ex. Find the slope

(a). $y = 3x - 4$

$y = mx + b$

$\uparrow$ slope * the number infront of x is the slope.

$m = 3$.

(b). $4x - 3y = 0$.

$\frac{-3y}{-3} = \frac{-4x}{-3}$

$y = \frac{4}{3}x$ $m = \frac{4}{3}$

ex. Find the inverse $f^{-1}(x)$

(a). $f(x) = 3x^3 - 1$

$y = 3x^3 - 1$

(swap) $x = 3y^3 - 1$

$\frac{x+1}{3} = \frac{\cancel{3}y^3}{\cancel{3}}$

$\frac{x+1}{3} = y^3$

$\sqrt[3]{\frac{x+1}{3}} = y$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{3}}$

(b). $f(x) = \frac{3x+1}{2}$

$y = \frac{3x+1}{2}$

(swap) $x = \frac{3y+1}{2}$

$2x = \frac{3y+1}{2}$

$2x = 3y + 1$

$\frac{2x-1}{3} = \frac{3y}{3}$

$y = \frac{2x-1}{3}$

$f^{-1}(x) = \frac{2x-1}{3}$

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ex. Simplify

(a). $2^3 \cdot 4^{-1} \cdot 8 \cdot (2^{-1})^2$

$= 8 \cdot 1/4 \cdot 8 \cdot 2^{-2}$

$= 2 \cdot 8 \cdot 1/2^2 = 2 \cdot 8 \cdot 1/4$

$= 16(1/4) = \boxed{4.}$

(b). $e^x(e^{2x} + e^4)$

$- 3\overset{0}{\cancel{\ln}(1)} - 2e^{4x}$

$+ \ln(e^3)$.

$= e^{3x} + e^{x+4} - 0 - 2e^{4x}$

$+ 3\underbrace{\ln(e)}_{=1}$

$= e^{3x} + e^{x+4} - 2e^{4x} + 3$

(c). $\dfrac{4x+4}{x^2-1}$

$x^2 + 1) \neq 0 = -1$

$= \dfrac{4x+4}{(x+1)(x-1)}$

$= \dfrac{4\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$= \dfrac{4}{x-1}$

* Rules :-

$a^{-1} = 1/a$

$(a^x)^y = a^{xy}$

$a^{-x} = 1/a^x$

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Chapter 2 - limits.

We write

$f(x) \to L$ as $x \to c$

<u>or</u>

$\lim_{x \to c} f(x) = L$

where L is a single number, if $f(x)$ is close to L, when x is close to c (but not equal to c).

ex. Find the limit

(a). $\lim_{x \to 2} f(x) = 2(2)+1 = 5.$

where $f(x) = 2x+1$

(b). $\lim_{x \to 1} \frac{x^2-1}{x-1}$ * we cant use ① because it will give an error, therefor you can use numbers like $\boxed{1.1}$ or $\boxed{0.99}$

* factor.

$= \lim \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 2} x+1 = 2$

(c). $\lim_{x \to 0} \frac{|x|}{x}$ → absolute value (if theres a negative sign, ignore it.)

$\frac{|0|}{0}$ = error !!!

- if x is bigger than 0 → 1.
- if x is less than 0 → -1.

$x > 0$ ex. 0.0001 $\frac{|0.0001|}{0.0001} = 1$

If x was a negative number. ex. -0.1

$\frac{|-0.1|}{-0.1} = \frac{0.1}{-0.1} = \boxed{-1}$

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(d). $\lim_{x \to 1} \frac{2x-2}{x^2+2x-3} = \lim_{x \to 1} = \frac{2(\cancel{x-1})}{(x+3)(\cancel{x-1})}$

$\lim_{x \to 1} = \frac{2}{x-3} = \frac{2}{4} = \frac{1}{2}.$

One sided limit

$\lim_{x \to c^+} f(x) = L$

if $f(x) \approx L$ when x is close to c, but x is ~~less~~ greater than c.

$\lim_{x \to c^-} f(x) = L$ x is close to c, but x is less than c.

ex. $f(x) = \begin{cases} x^2+1, & x > 2 \\ x-1, & x \cancel{<} 2 \end{cases}$

(a). $\lim_{x \to 2^+} f(x) = 2^2+1 = 5.$

(b). $\lim_{x \to 2^-} f(x) = 2-1 = 1.$

(c). $\lim_{x \to 2} f(x) =$ does not exist, because its not specified!

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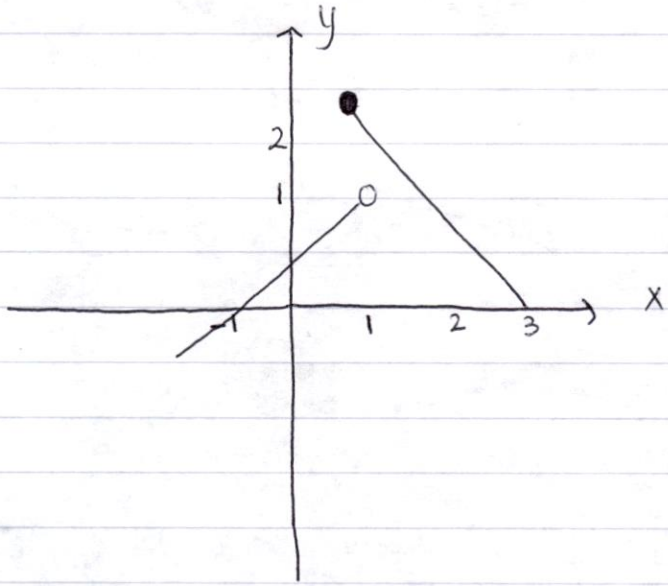
ex.



(a). $\lim_{x \to 1^-} f(x) = 1.$

(b). $\lim_{x \to 1^+} f(x) = 2.$

(c). $\lim_{x \to 1} f(x) =$ does not exist, because the limit is not specified.

Rule = If $\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x)$

$\Rightarrow \lim_{x \to c} f(x) =$ does not exist. (DNE).

(d). $\lim_{x \to -1} f(x) = 0.$

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Rule : if $\lim_{x \to c} f(x) = 0$

and

$\lim_{x \to c} g(x) = 0$

$\Rightarrow \lim_{x \to c} \frac{f(x)}{g(x)}$ is called $\frac{0}{0}$ interdeterminate

the limit may not exist or may exist

$\Rightarrow$ further investigation

ex. Is it 0/0 interdeterminate?

(a). $\lim_{x \to 0} \frac{2x^2}{x}$ = yes

(b). $\lim_{x \to 0} \frac{e^{x^2} - 1}{ex^2 + 1} = \frac{1-1}{1+1} = \frac{0}{2}$ = NO.

(c). $\lim_{x \to 5} \frac{2^{x-5} - 1}{2x - 10} = \frac{0}{0}$ = yes

Rule : if $\lim_{x \to c} g(x) = 0$

but

$\lim_{x \to c} f(x) \neq 0$

$\Rightarrow \lim_{x \to c} \frac{f(x)}{g(x)}$ = does not exist

ex. $\lim_{x \to 0} \frac{2x+1}{x^3}$

not zero

$1 \rightarrow x \rightarrow 0$

zero

$0 \rightarrow x \rightarrow 0$

$\frac{1}{0}$ = does not exist.

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Student 1
Student 2
Student 3
Student 4

Students final grades example.

Q1. What is my location at 11:00 am? → function (exact).
in class

Q2. What is my location around 11:00 am? → limit (behaviour).
Before: Office. After: In Class. Behaviour: DNE

Q3. What is my location at 11:30 am? → function
in class

Q4. What is my location around 11:30 am? → limit.
Behaviour: in class. Before: in class. After: in class.

Example:

$$f(x) = x^2+4x+1$$
$$f(2) = 2^2+4(2)+1 = 4+8+1 = 13.$$

Example:

$$\lim_{x \to 2} x^2+4x+1 = 2^2+4(2)+1 = 13$$

Behaviour around 2.

Behaviour before: left limit $\lim_{x \to 2^-} x^2+4x+1$
Behaviour After: right limit $\lim_{x \to 2^+} x^2+4x+1$

- Steps how to calculate the limit:

Step 1. We plug the number inside.
Step 2. If it is a nice number we are done. * no error numbers.
Step 3. If we have $\frac{0}{0}$ we factor, cancel and go back to step 1.
cancellation is always there.

infinite limits
vertical asymptote

Step 4. if $\frac{4}{0}$, we look left limit and right limit.
left limit = $\lim_{x \to 4^-} f(x) = +\infty$ or $-\infty$ → if left = right then that will be limit
right limit = $\lim_{x \to 4^+} f(x) = +\infty$ or $-\infty$ → if left ≠ right then lim DNE
Step 5. We have to decide who is dominating. bottom / left (top) or same power.

horizontal asymptote

if top is dominating = lim → $+\infty$ / $-\infty$ if bottom dominating = lim → 0.
if same power = lim → division of leading coefficient.

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Step6. Piecewise functions.

Step7. Continuos. → no hole, no jump, no break. →

$$\left.\begin{array}{l} f(x) \\ \lim_{x\to a^-} f(x) \\ \lim_{x\to a^+} f(x) \\ \lim_{x\to a} f(x) \end{array}\right\} \text{all equal}$$

How to decide who is diminating ?

- if we have x^n, the larger n the more power.
- e^x is more powerful it is $e^{+\infty}$
- $e^x = 0$ if $e^{-\infty}$
- ln x weaker than everything.

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Example 1.

step3 → calculate top and bottom seperate on the calculator

$$\lim_{x\to 1} \frac{x^2-3x+2}{x^2-x} = \frac{0}{0} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}x} = \frac{-1}{1} = -1$$

↳ You can find the x number from the lim only when $\frac{0}{0}$ occurs
example:
$\lim_{x\to 2} \rightarrow (x-2)$
$\lim_{x\to -3} \rightarrow (x+3)$

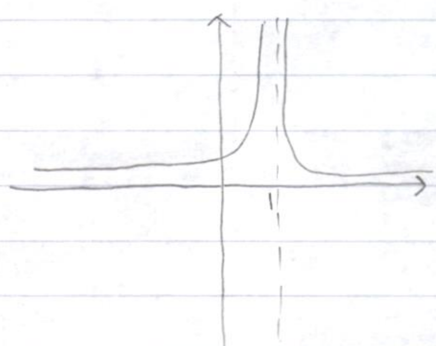
Example 2.

step4

$$\lim_{x\to 1} \frac{x+2}{(x-1)^2} = \frac{3}{0} = \cancel{\frac{0.9+2}{(0.9-1)^2} = \frac{+}{+}} = +\infty$$

$$\lim_{x\to 1^-} \frac{x+2}{(x-1)^2} = \frac{0.9+2}{(0.9)-1)^2} = \frac{+}{+} = +\infty$$

$$\lim_{x\to 1^+} \frac{x+2}{(x-1)^2} = \frac{1.1+2}{(1.1-1)^2} = \frac{+}{+} = +\infty$$



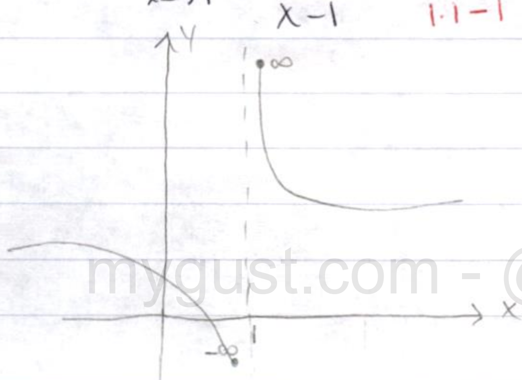
Example 3.

step4

$$\lim_{x\to 1} \frac{x+2}{x-1} = \frac{3}{0} =$$ does not exist because left limit is not equal to right limit.

$$\lim_{x\to 1^-} \frac{x+2}{x-1} = \frac{0.9+2}{0.9-1} = \frac{+}{-} = -\infty$$ * Choose a number less than 1 for left limit.

$$\lim_{x\to 1^+} \frac{x+2}{x-1} = \frac{1.1+2}{1.1-1} = \frac{+}{+} = +\infty$$ * Choose a number more than 1 for right limit.



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limits at the infinity.

Step 5

$$\lim_{x \to +\infty} \frac{x^2 + 4x}{x^3 - 3}$$

more powerful

$$\lim_{x \to -\infty} \frac{x^3 - 7x + 1}{17x^2 + 5x + 1}$$

more powerful

1. Bottom

$$\lim_{x \to \infty} \frac{\boxed{x^2} - 4x}{\boxed{x^3} + 5x + 1} = 0$$

horizontal asymptote.
$y = 0$.

2. Top

$$\lim_{x \to +\infty} \frac{\boxed{x^3} + 5x + 1}{\boxed{x^2} - 4x} = \frac{+}{+} = +\infty$$

x

* you see the sign infront of the x number

3. Bottom

$$\lim_{x \to -\infty} \frac{\boxed{x^2} - 4x}{\boxed{x^3} + 5x + 1} = 0$$

$y = 0$.

4. Same power

$$\lim_{x \to \infty} \frac{\boxed{3x^4} + 5x^3}{\boxed{7x^4} - 9x + 1} = \frac{3}{7}.$$

$y = \frac{3}{7}$

5. Top

$$\lim_{x \to \infty} \frac{4 - \boxed{3x^2}}{\boxed{9x} - 1} = \frac{-}{+} = -\infty$$

x

6. Top

$$\lim_{x \to -\infty} \frac{4 - \boxed{3x^2}}{\boxed{9x} - 1} = \frac{-}{-} = +\infty$$

x

7. Top.

$$\lim_{x \to \infty} \frac{\boxed{5x^3} + 4}{1} = +\infty$$

hidden Bottom → 1

x

8. Same power

$$\lim_{x \to \infty} \frac{5x^{-3} + 4x^0}{1x^0} = 4$$

$y = 4$

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put (+) its (+).

TOP $\lim_{x \to \infty} \dfrac{\boxed{e^x}+4}{\boxed{x^2}+9} = +\infty$

more powerful.

Bottom $\lim_{x \to -\infty} \dfrac{e^x + \boxed{4}}{\boxed{x^2}+9} = 0.$

put (-), it will be (+), because $-/- = +$.

TOP $\lim_{x \to -\infty} \dfrac{\boxed{e^{-x}} + 7}{\boxed{x} - 5} = \dfrac{+}{-} = -\infty$

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$$\lim_{x\to\infty} \frac{\boxed{e^x}+x^2}{x^3-\boxed{x^4}} = \frac{+}{-} = -\infty$$

TOP

$$\lim_{x\to-\infty} \frac{e^x+\boxed{x^2}}{x^3-\boxed{x^4}} = 0$$

Bottom

$$\lim_{x\to\infty} \frac{e^{-x}+\boxed{4x}}{\boxed{3x}+5} = \frac{4}{3}$$

Same

$$\lim_{x\to-\infty} \frac{\boxed{e^{-x}}+4x}{\boxed{3x}+5} = \frac{+}{-} = -\infty$$

TOP

$$\lim_{x\to\infty} \frac{e^{-x}+\boxed{4}}{\boxed{1}} = \frac{4}{1}$$

Same

hidden 1

$$\lim_{x\to-\infty} \frac{\boxed{e^{-x}}+4}{\boxed{1}} = \frac{+}{+} = +\infty$$

TOP

hidden 1

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less than or equal.

$$f(x) \begin{cases} x+4 & x \leq -1 \\ x^2+2 & -1 \leq x < 2 \\ x+1 & 2 < x \end{cases}$$

less than

Border points.
$x = -1$
and
$x = 2$

$f(-2) = -2+4 = 2$
$f(-1) = -1+4 = 3.$
$f(0) = 0^2+2 = 2$
$f(2) =$ undefined.
$f(3) = 4.$

$\lim_{x \to 2^-} f(x) = 2$

$\lim_{x \to 1^-} f(x) = 3$

$\lim_{x \to 0} f(x) = 2$

$\lim_{x \to +3} f(x) =$ DNE

$\lim_{x \to -1^-} f(x) = 4$

$\lim_{x \to -1} f(x) = 3$

$\lim_{x \to -1}{}^{+} f(x) = 3$

$\lim_{x \to 2}{}^{-} f(x) = 6$

$\lim_{x \to 2}{}^{+} f(x) = 3$

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Step 6.

$$f(x) \begin{cases} 4x+1 & x < 1 \\ 3x+b & x \geq 1 \end{cases}$$

border point = 1.

Find:

$\lim_{x \to 0} f(x) = 4(0)+1 = 1$

$\lim_{x \to 2} f(x) = 3(2)+b = 6+b$

$\lim_{x \to 1^-} f(x) = 4(1)+1 = 5$

$\lim_{x \to 1^+} f(x) = 3(1)+b = 3+b$.

- Can we find $\lim_{x \to 1} f(x)$?

NO.

because we cannot calculate and we dont know if its equal

$$f(x) \begin{cases} 5x^2+2 & x < 1 \\ 3x+b & x \geq 1 \end{cases}$$

a) $\lim_{x \to 0} f(x) = 5(0)^2+2 = 2$

b) $\lim_{x \to 3} f(x) = 3(3)+b = 9+b$

c). find b so that $f(x)$ is continous at $x=1$.

$5(1)^2+2 = 7$

$3(1)+b = 3+b$

$7 = 3+b$

$b = 7-3 = 4$

$b=4$

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Derivative (differentiate).

seen in college algebra $y = f(x) \longrightarrow$ function

in calculus $y' = f'(x) = \frac{dy}{dx} \longrightarrow$ first derivative

$y'' = f''(x) \longrightarrow$ second derivative

$y''' = f'''(x) \longrightarrow$ third derivative.

note:

$y^{(4)}$ = fourth derivative

y^4 = fourth power.

* with bracket means (derivative), without bracket means power.

Exam! $\boxed{\text{Rule 1:}}$ definition of the derivative or the 4 step method.

$$y' = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$$ * memorize the formula!

Rule 2: power rule

$$(x^n)' = n \cdot x^{n-1}$$

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Example 1.

Find the derivative of $\boxed{y = x^2}$ using the definition of the derivative. (4 step method).

$y = x^2$

$y' = 2x^{2-1} = 2x^1 = 2x \longrightarrow$ Rule 2. $(x^n)' = n \cdot x^{n-1}$

Find the derivative of $\boxed{y = x^2 + 3x^1}$ Using the 4 step method.

$y' = 2x + 3 \cdot x^{1-1} \longrightarrow$ Rule 2.

$= 2x + 3x^0$

$= 2x + 3$

Find the derivative of $\boxed{y = \frac{1}{x}}$ using the 4 step method.

$y' = -1 \cdot x^{-1-1} = -1 \cdot x^{-2} = \boxed{\frac{-1}{x^2}} \longrightarrow$ Rule 2

Optional to write

Find the derivative of $\boxed{y = \sqrt{x}}$ using the 4 step method.

$y' = \sqrt{x} = x^{\frac{1}{2}} \longrightarrow$ Rule 2.

$y' = \frac{1}{2} x^{\frac{1}{2} - 1}$

$= \frac{1}{2} x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$ optional to write

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$\longrightarrow$ will be change ex. $y = x^3$

$y = x^2$ * coming in exam

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$ Rule 1

$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= 2x + 0$$

$$= 2x$$

$y = x^2 + 3x$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + 3h - \cancel{x^2} - \cancel{3x}}{h}$$ Rule 1

$$= \lim_{h\to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}}$$

$$= \cancel{\lim}\ 2x + 3$$

$y = \frac{1}{x}$

Rule 1

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1(x)}{x+h\,(x)} - \frac{1(x+h)}{x\,(x+h)}}{h}$$ - multiply common denominator.

$$= \lim_{h\to 0} \frac{\frac{x}{(x+h)\cdot x} - \frac{x+h}{(x+h)\cdot x}}{h}$$

$\longrightarrow$ continue.

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$$= \lim_{h \to 0} \frac{\frac{\cancel{x} - \cancel{x} - h}{(x+h) \cdot x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{(x+h) \cdot x}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h} \cdot (x+h) \cdot x}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h) \cdot x}$$

$$= \lim_{h \to 0} \frac{-1}{x \cdot x}$$

$$= \frac{-1}{x^2}$$

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* if you see a square root → this is the method.

Find the derivative of $\boxed{f(x) = \sqrt{x}}$ using the 4 step method.

$f(x) = \sqrt{x}$

$$y' \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\overset{a}{\sqrt{x+h}} \overset{-}{-} \overset{b}{x}}{h} \cdot \frac{\overset{a}{\sqrt{x+h}} \overset{+}{+} \overset{b}{\sqrt{x}}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h \cdot (\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x}} = \frac{1}{2x^{1/2}}$$

Rule:

$(a-b)(a+b) = a^2 - b^2$

$(\sqrt{x+h}) = x+h$

$(\sqrt{x})^2 = x$

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Rule 2:

$(x^n)' = n \cdot x^{n-1}$

$(7)' = 7x^6$

$\left(\frac{1}{x^3}\right)' = (x^{-3}) = -3x^{-4}$

Rule 3: ↱ constant number.

$(c)' = 0$ * if you didn't see x, number is 0.

$(0.5)' = 0$

$(2)' = 0$ $(e^2) = 0.$

Rule 4:

$(x^n + c)' = n \cdot x^{n-1}$ $(x^3+5)' = 3x^2$

Rule 5:

$(c \cdot x^n)' = c \cdot n \cdot x^{n-1}$ $(5x^3)' = 5 \cdot 3 \cdot x^2 = 15x^2$

Rule 6:

$(e^x)' = ex$

$(2^x)' = 2^x \cdot \ln^2$

$(7^x)' = 7^x \cdot \ln 7$

$\ln x = \log_e x$ $(\ln x)' = 1/x$

$\ln e = 1$ $(\log_2 x)' = 1/x \cdot \ln 2$

$(\log_9 x)' = 1/x \cdot \ln 9$

Rule 7: Product rule (multiplication)

$(f(x) \cdot g(x))' = f'(x) g(x) + f(x) g'(x)$

$[(x^3+4x) \cdot e^x]' = (3x^2+4) \cdot e^x + (x^3+4x) e^x$

$\underset{f(x)}{\downarrow}$ $\underset{g(x)}{\downarrow}$

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Rule 8 : quotient rule (division)

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\left[\frac{\overbrace{x^3-3}^{f(x)}}{\underbrace{4x^2-7x}_{g(x)}}\right]' = \frac{3x^2 \cdot (4x^2-7x) - (x^3-3) \cdot (8x-7)}{(4x^2-7x)^2}$$

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$4x^{\frac{-1}{2}}$

2a. $f(x) = \frac{5}{\sqrt{x}} + \frac{4}{x^{2}} + 5^x - \ln(6)$

send things bottom to top and remove $\sqrt{\ }$

$= 5x^{-1/2} + 4x^{-2} + 3^x - \ln(6).$

$f'(x) = \frac{-5}{2} x^{-1/2-1} - 8x^{-2-1} + 5^x \cdot \ln(5) - 0$

$= \frac{-5}{2} x^{\frac{-3}{2}} - 8x^{-3} + 5^x \ln(5)$

c. $f(x) = \ln(x) \cdot (3x-2)$

0, because 2 is a number.

$f'(x) = \frac{1}{x} \cdot (3x-2) + \ln(x) \cdot (3-0)$

b. $f(x) = 4x^3 - \frac{4}{x^5} + x - \frac{4}{\sqrt{x}} + 4^6$

$= 4x^3 - 4x^{-5} + x - 4x^{\frac{-1}{2}} + 4^6$

$f'(x) = 12x^2 + 20x^{-6} + 1 + 2x^{\frac{-3}{2}} + 0$

Product rule

d. $f(x) = x^7 \cdot e^x$

$f'(x) = 7x^6 e^x + x^7 \cdot e^x$

e. $f(x) = \frac{\ln(x)}{3x-2}$

$f'(x) = \frac{\frac{1}{x} \cdot (3x-2) - \ln(x) \cdot (3-0)}{(3x-2)^2}$

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$$f(x) = \frac{2\ln(x)}{x^3 - 2}$$

$$f'(x) = \frac{\frac{2}{x} \cdot (x^3 - 2) - 2\ln(x)(3x^2)}{(x^3 - 2)^2}$$

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Chainrule: two functions come together.

Rule 9:

$$((f(x))^n)' = n \cdot f'(x) \cdot (f(x))^{n-1} \qquad (x^n)' = n \cdot x^{n-1}$$

Rule 10:

$$(e^{f(x)})' = f'(x) \cdot e^{f(x)} \qquad (e^x)' = e^x$$

Rule 11:

$$(\ln f(x))' = \frac{f'(x)}{f(x)} \qquad (\ln(x))' = \frac{1}{x}$$

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$(x^{20})' = 20x^{19}$

$(e^{x})' = e^{x}$

$(\ln(x))' = \frac{1}{x}$

$f(x) = \left((x^3 + x)^{20}\right)' = 20 \cdot (3x^2+1) \cdot (x^3+x)^{19}$

$f(x) = (e^{x^3+x})' = (3x^2+1)\, e^{x^3+3}$

$f(x) = \left(\ln(x^3+x)\right)' = \dfrac{3x^2+1}{x^3+x}$

4a. $f(x) = (3x^4 + 4)^5$

$= 5 \cdot (12x^3 + 0) \cdot (3x^4+4)^4$

4b. $f(x) = \ln(x^2 + 4x + 1)$

$= \dfrac{2x+4}{x^2+4x+1}$

4c. $f(x) = e^{x7 - \sqrt{x}}$

$= \left(3x^2 - \frac{1}{2}x^{\frac{-1}{2}}\right) \cdot e^{x3 - \sqrt{x}}$

4d. $h(x) = (x^4 + 5x)\ln(x^3 + 6x^2)$

$=$

4e. $h(x) = \ln(x^2 - 3)\, e^{(x2-3)}$ $\quad f' \cdot g + f \cdot g'$

$= \dfrac{2x}{x^2-3} \cdot e^{x2-3} + \ln(x^2-3)\, 2x e^{x2-3}$

$f' \quad g \quad + \quad f \quad \cdot \quad g'$

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4f. $f(x) = x^4 \cdot e^{x4}$

$= 4x^3 \cdot e^{x4} + x^4 \cdot e^{x4}$ $4x^3$

$f' \quad g \quad + f \quad g'$

4g. $g(x) = \dfrac{3x^2 \; f}{(x^2+5)^3 \; g}$ caution rule. $\dfrac{f' \cdot g - f \cdot g'}{g^2}$

$f' \quad g \quad f \quad g'$

$= \dfrac{6x \cdot (x^2+5)^3 - 3x^2 \cdot 3 \cdot 2x \cdot (x^2+5)^{3-1}}{((x^2+5)^3)^2}$

4h). $l(x) = \dfrac{\sqrt{x^3+3x-1}}{x^2}$

$= \dfrac{(x^3+3x^{-1})^{\frac{1}{2}}}{x^2}$ ① re-write the function

$f' \quad g \quad f \quad g'$

$= \dfrac{\frac{1}{2} \cdot (3x^2+3)(x^3+3x-1)^{\frac{1}{2}-1} \cdot x^2 - (x^3+3x^{-1})^{\frac{1}{2}} \cdot 2x}{(x^2)^2 \quad g^2}$

4i. $f(x) = (\ln(x))^7$ Rule 9.

$= 7 \cdot \frac{1}{x} \cdot (\ln(x))^{7-1}$

$(\ln(x^2+x))^7$ Rule 9 and 11.

$= 7 \cdot \dfrac{2x+1}{x^2+x} \cdot (\ln(x^2+x))^{7-1}$

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double chain rule.

Rule 10. then 9.

$$e(^{x2+\sqrt{x}})^{11} = 11\cdot\left(2x+\tfrac{1}{2}x^{\frac{-1}{2}}\right)\cdot\left(x^2+\sqrt{x}\right)^{10}$$

Rule 10 then 9.

$$\left(e^{x2+\sqrt{x}}\right)^{21} = 21\cdot\left(2x+\frac{1}{2}x^{\frac{-1}{2}}\right)e^{x^2\sqrt{x}}\cdot\left(e^{x2+\sqrt{x}}\right)^{20}$$

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college algebra

Rule 12: tangent line ↱ point ↱ slope

equation of line (x_0, y_0) $m = f'(x_0)$

↑ for the tangent line

$y - y_0 = m(x - x_0)$

↑ equation of line

Example: find equation of the tangent line to the function

$y = x^2 + 4x - 2$ at the point $(1, 3)$.

$f'(x) = 2x + 4$ x_0 ↲ ↳ y_0

Find slope $2(1) + 4 = 6$ * plug (1) to the formula

$y - y_0 = m(x - x_0)$

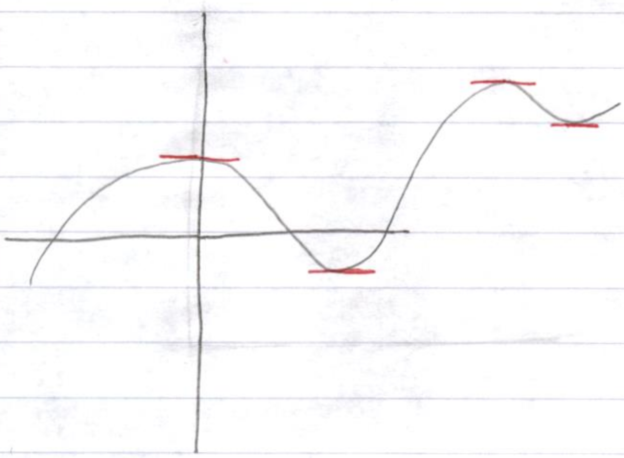
$y - 3 = 6(x - 1)$

tangent line is line touching to the function at a point.

Rule 13: horizontal tangent line

$m = 0$

$f'(x) = 0$



On the graph, what are the points where we have horizontal tangent line?

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For the function $f(x) = x^2 + 4x - 2$ find the points where the tangent line is horizontal.

Step1 = $m = 0$.

Step2 = $f'(x) = 2x + 4 = m$

Step3 = $2x + 4 = 0$

$x = -2$

$= (x^2 + 4x - 2)^{\frac{1}{2}}$

For the function $f(x) = \sqrt{x^2 + 4x - 2}$ find the points where the tangent line is horizontal.

Step1 = $m = 0$

Step2 = $f'(x) = \frac{1}{2}(x^2 + 4x - 2)^{\frac{1}{2}} \cdot (2x + 4)$

solve

college algebra Step3 = $\frac{1}{2}(x^2 + 4x - 2)^{\frac{1}{2}} \cdot (2x + 4) = 0$

$\frac{2x + 4}{2(x^2 + 4x - 2)^{1/2}} = \frac{0}{1}$

* send negative numbers down.

* Cross multiply

$2x + 4 = 0$

$x = -2$.

For the function $f(x) = e^{x2 + 4x - 2}$ find the points where the tangent line is horizontal.

Step1 = $m = 0$

Step2 = $f'(x) = (2x + 4)\, e^{x2 + 4x - 2}$

Step3 = $(2x + 4) e^{x2 + 4x - 2} = 0$

$2x + 4 = 0$ or $e^{x2 + 4 - 2} = 0$. ← if the number is not 0, You take log.

$\boxed{x = -2}$

= no solution !

note; e to the power anything

$e^{\cdots} > 0$.

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Rule 14: implicit differentiation

$$\begin{cases} (x)' = 1 \\ (y)' = 1 \cdot \dfrac{dy}{dx} \end{cases}$$

Example: find $\dfrac{dy}{dx} = y'$ $\quad x^3 + y^4 + 9x = 25$

$$3x^2 + 4y^3 \cdot \frac{dy}{dx} + 9 = 0$$

$$4y^3 \cdot \frac{dy}{dx} = -3x^2 - 9$$

$$\frac{dy}{dx} = \frac{-3x^2 - 9}{4y^3}$$

Review paper Q6

$$x^2 + \underbrace{3x \cdot y}_{\text{product rule.}} + 4y^2 = 11$$

a) Find $\dfrac{dy}{dx}$

b). find tangent line $(\overset{x_0}{-1}, \overset{y_0}{2})$.

$$2x + 3 \cdot y + 3x \cdot 1 \cdot \frac{dy}{dx} + 8y \cdot \frac{dy}{dx} = 0$$

$$3x \cdot \frac{dy}{dx} + 8y \cdot \frac{dy}{dx} = -2x \mp 3$$

$$\frac{dy}{dx}(3x + 8y) = -2x - 3$$

$$\frac{dy}{dx} = \frac{-2x - 3}{3x + 8y}$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{-4}{13}(x + 1)$$

* if there is no $\dfrac{dy}{dx}$ infront you take it → and it becomes

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Q7. Product rule

$x^2y - y^3 = 6$

$2x \cdot y + x^2 \cdot 1 \cdot \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$

$x^2 \cdot 1 \cdot \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = -2x \cdot y$

$2x=0$

$\frac{dy}{dx}(x^2 - 3y^2) = -2x \cdot y$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 3y^2}$ ← Find m here.

$m = \frac{-4}{1-12} = \frac{4}{11}$

tangent line

$y - y_0 = m(x - x_0)$

$y + 2 = \frac{4}{11}(x+1)$

Q8. Rule #14 chain rule 10. chain rule

product rule ← $xy^5 + e^{x-1} + e^y = 2$ tangent line $(1, 0)$ x_0 y_0

$1 \cdot y^5 + x \cdot 5y^4 \cdot \frac{dy}{dx} + 1 \cdot e^{x-1} + 1 \cdot \frac{dy}{dx} e^y = 0$

$x \cdot 5y^4 \cdot \frac{dy}{dx} + e^y \frac{dy}{dx} = -y^5 - e^{x-1}$

$\frac{dy}{dx}(x \cdot 5y^4 + e^y) = y^5 - e^{x-1}$

$\frac{dy}{dx} = \frac{-y^5 - e^{x-1}}{x \cdot 5 \cdot y^4 + e^4}$

$y - y_0 = m(x - x_0)$

$y - 0 = -1(x - 1)$

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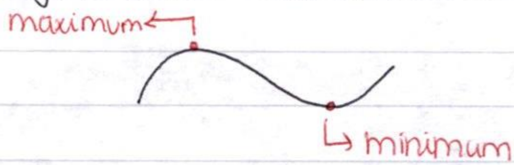
Youtube chanel : PatrickJMT ↑go up ↓ go down

First derivative test
1. find intervals of increase and decrease.
2. find local maximum and minimum.



* the numbers are called
Critical numbers.

- You find the derivative, and clean.
- Put the derivative = 0 and solve * most difficult step. * critical number
- Sign chart * easiest step.
- tell if local maximum or minimum

10 points.

* $f(x) = (x+3)\cdot(x-2)^2$

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$f(x) = (x^2-1)^3$

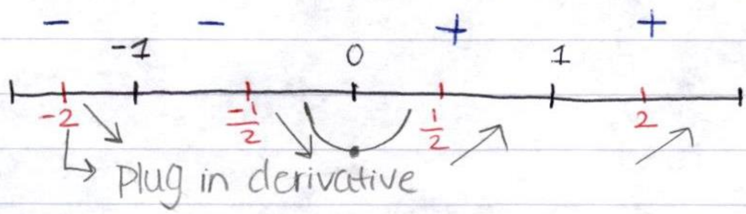
$f'(x) = 3(x^2-1) \cdot (2x)$ derivative.

$= (6x) \cdot (x^2-1)^2$

$6x = 0 \quad\quad x^2-1=0$

$\quad\quad\quad (x-1)(x+1)=0$

$\boxed{x=0} \quad \boxed{x=1} \quad \boxed{x=-1}$



$(-\infty, -1)$ decrease
$(-1, 0)$ decrease
$(0, 1)$ increase
$(1, \infty)$ increase

$x = -1$ no change
$x = 0$ local minimum
$x = 1$ no change.

Product rule.

$f(x) = (x-2)^2(x+3)$

$f'(x) = 2(x-2) \cdot 1 \cdot (x+3) + (x-2)^2 \cdot 1$

$= (x-2)\,[2(x+3) + (x-2)]$

$= (x-2)\,[2x+6+x-2]$

$= (x-2)(3x+4) = 0$

$\boxed{x=2} \quad\quad 3x+4=0$

$3x = -4$

$\boxed{x = \frac{-4}{3}}$ * critical numbers

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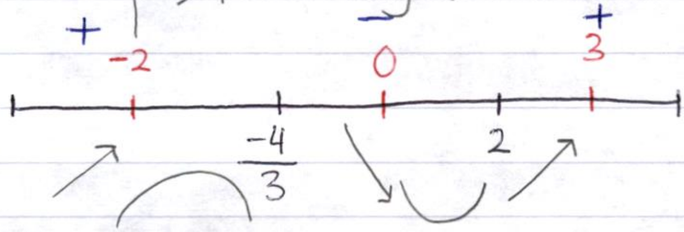
find middle point

put in any of the derivatives.



$(-\infty, -\frac{4}{3})$ increase
$(-\frac{4}{3}, 2)$ decrease
$(2, \infty)$ increase

$x = -\frac{4}{3}$ local maximum

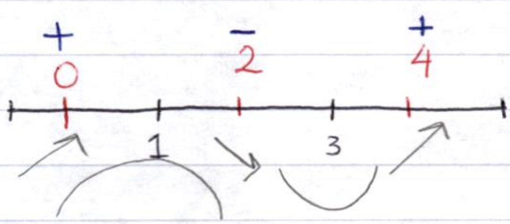
$x = 2$ local minimum

math 130 - review 2.

1a. $f(x) = x^3 - 6x^2 + 9x + 12$

$f'(x) = 3x^2 - 12x + 9$
$= 3(x^2 - 4x + 3)$
$= 3(x-3)(x-1)$

$\boxed{x=3}$ $\boxed{x=1}$



$(-\infty, 1)$ increase
$(1, 3)$ decrease
$(3, \infty)$ increase

$x = 1$ local maximum
$x = 3$ local minimum

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$\ulcorner\!\rightarrow$ product rule

$f(x) = (x+2)^2 (x-3)$

$f'(x) = 2(x+2) \cdot 1 \cdot (x-3) + (x+2)^2 \cdot 1$

$= (x+2)[2(x-3) + (x+2))$

$= (x+2)[2x - 6 + \cancel{2}x + 2]$

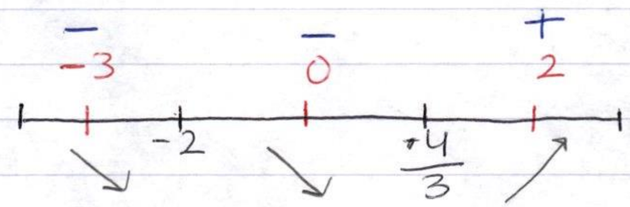
$= (x+2) \; 3x - 4 = 0$

$\downarrow \qquad \downarrow$

$\boxed{x = -2}$ $\qquad 3x - 4 = 0$

$3x = 4$

$\boxed{x = \frac{4}{3}}$



$(-\infty, -2)$ decrease
$(-2, 0)$ decrease
$(0, 2)$ increase

$f(x) = x^3 - 3x^2$

In $\qquad 2(x+2)(x-3) = 0$

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Before: first derivative → increase / decrease, local max / min

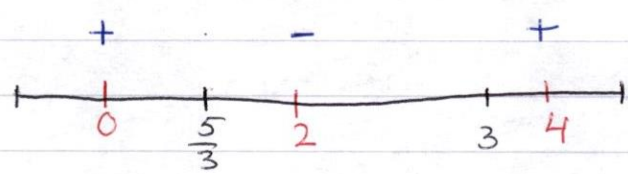
Now : Second derivative test → concave up / down.

increase	$f'>0$, $f''>0$	$f'>0$, $f''<0$	$f'>0$, $f''=0$
decrease	$f'<0$, $f''>0$	$f'<0$, $f''<0$	$f'<0$, $f''=0$
	concave up	concave down	line

$$f(x) = (x-1)(x-3)^2$$

$$\begin{aligned} f'(x) &= x(x-3)^2 + (x-1)\cdot 2\cdot 1\cdot (x-3) \\ &= (x-3)\left[(x-3) + 2(x-1)\right] \\ &= (x-3)\left[x-3+2x-2\right] \\ &= (x-3)(3x-5) = 0. \end{aligned}$$

critical numbers. → $x=3$ $\quad x=\frac{5}{3}$



$(-\infty, \frac{5}{3})$ inc
$(\frac{5}{3}, 3)$ dec
$(3, \infty)$ inc

from + to - local max $\quad x = \frac{5}{3}$ local maximum

from - to + local min $\quad x = 3$ local minimum

from + to + no change.

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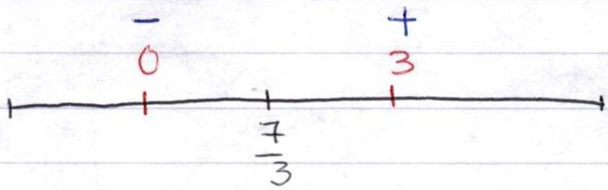
* take the derivative of the last one - product-rule.

$f''(x) = 1 \cdot (3x-5) + (x-3) \cdot 3$

$= 3x - 5 + 3x - 9$

$= 6x - 14 = 0.$

Inflection points $\rightarrow x = \frac{14}{6} = \frac{7}{3}$



$(-\infty, \frac{7}{3})$. concave down

$(\frac{7}{3}, \infty)$. concave up.

* to find absolut min/max. write the critical numbers and points.

plug in the original function

$3 = (3-1)(3-3)^2 = 0.$ absolute minimum

~~$\frac{5}{3}$ 1.6~~ You cross because it's outside 2 and 4. they should be between the given points

$2 = (2-1)(2-3)^2 = 1$

$4 = (4-1)(4-3)^2 = 3$ absolute maximum

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$f(x) = x^3 - 6x^2 + 9x + 12$

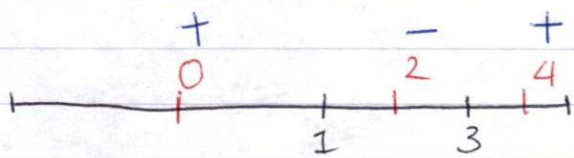
$f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$ take for 2nd derivative because easiest.

$= 3(x-3)(x-1) = 0.$

$\downarrow \quad \downarrow$

Critical numbers $x = 3 \quad x = 1$



$x = 1$ local maximum

$x = 3$ local minimum

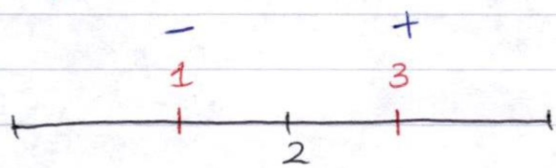
$(-\infty, 1)$ increase

$(1, 3)$ decrease

$(3, \infty)$ increase.

$f''(x) = 6x - 12 = 0$

$x = \frac{12}{6} = 2.$ inflection point



$(-\infty, 2)$ concave down

$(2, \infty)$ concave up.

$f(2) = 14$ absolute maximum

$f(3) = -42$ absolute minimum

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1 $\lim_{x \to 1} \frac{x^2-1}{x^2-3x+2} = \frac{0}{0}$

$\lim_{x \to 1} \frac{2x}{2x-3} = \frac{2(1)}{2(1)-3} = \frac{2}{2-3} = \frac{2}{-1} = -2$

2 $\lim_{x \to 1} \frac{x \ln x}{1-e^{x-1}} = \frac{0}{0}$

$\lim_{x \to 1} \frac{\ln(x) + x \cdot \frac{1}{x}}{-e^{x-1} \cdot (1)} = \frac{\ln(1) + (1) \cdot \frac{1}{(1)}}{-e^{1-1} \cdot (1)} = \frac{0+1 \times 1}{-e^0}$

$= \frac{1}{-1} = -1$

3 $\lim_{x \to 1} \frac{x^5-3x^2+2}{\ln x} = \frac{0}{0}$

$\lim_{x \to 1} \frac{5x-6x}{\frac{1}{x}} = \frac{5(1)-6(1)}{\frac{1}{1}} = \frac{5-6}{1} = \frac{-1}{1} = -1$

4 $\lim_{x \to 0} \frac{e^x-1}{x^2} = \frac{0}{0} = \frac{e^x \cdot (1) - 0}{2x} = \frac{e^1 \cdot (1)}{2(0)} = \frac{1}{0}$

Step 4

left limit $\lim_{x \to -0.1} \frac{e^{-0.1}-1}{(-0.1)^2} = \frac{-}{+} = -\infty$

right limit $\lim_{x \to 0.1} \frac{e^{0.1}-1}{(0.1)^2} = \frac{+}{+} = +\infty$

DNE

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Interest rate

1. 1000 KD invested in bank, compounded continuously for 6 years with 7% ratio. How much will be in the bank after the period.

$P = 1000 \quad r = 0.07 \quad t = 6 \quad A = ?$

$A = Pe^{rt} \quad A = 1000e^{(0.07)(6)} = 1521.96$

2. money invested in bank with 10% rate compounded continously. how long will it take for money to double.

$r = 0.01 \quad A = 2p \quad p = p \quad t = ?$

$2\cancel{p} = \cancel{p}e^{rt}$

$\ln 2 = 0.1(t) \quad t = \frac{\ln(2)}{0.1} = 6.93$ yr.

3. 1000 KD invested in bank compounded continously for 3 year and at the end of period 1300 KD is the account. what was the rate.

$P = 1000 \quad A = 1300 \quad r = ? \quad t = 3$

$A = Pe^{rt}$

$\frac{1300}{1000} = \frac{1000e^{r(3)}}{1000} \qquad r = \frac{\ln(1.3)}{3} = 0.08$

$1.3 = e^{r(3)} \qquad r = 0.08 \times 10 = 8.1$

$\ln(1.3) = r(3)$

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marginal analysis.

$C(x) = 176x\ 1.5x \ = C'(x) = 1.5$

x = how many
p = price of
$r(x)$ = revenue $\rightarrow p \cdot x$
$c(x)$ = cost
$p(x)$ = profit $\rightarrow r(x) - c(x)$

$(x)(15 - 0.02x)$

$f' = 1 \qquad s' = 0 - 0.02$

$1 \cdot 15 - 0.02x$

$C'(x)$

$P = 3x - 0.02x^2 - 292 - 0.1x$

$P'(x) = 3 - 0.04x - 0 - 0.1$

$P'(x) = 0$

$P'(x) = 29$

average $C(x) = \dfrac{C(x)}{x} = \dfrac{123 + 6.8x}{x}$

$C'(x) = 0x(3.6)$

$P'(x) = 3 - 0.16x$

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Intergal.

what is the derivative of $(x^3)'$? derivative

- $3x^2$

which function has derivative $3x^2$? anti-derivative / intergal

- x^3 + constant number (c). $\int 3x^2\,dx$

1. $\int x^3 + \frac{1}{x^2} + \frac{1}{x} + \sqrt{x} - 4x^0\,dx$ (x^{-2}, $x^{\frac{1}{2}}$)

$$\frac{x^{3+1}}{3x+1} + \frac{x^{-2+1}}{-2+1} + \ln|x| + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \frac{-4x^{0+1}}{0+1} + C$$

2. $\int 4x^2 + 3x^1 - 7x^0 + \frac{1}{\sqrt{x}}\,dx$ ($x^{-\frac{1}{2}}$)

$$\frac{4x^{2+1}}{2+1} + \frac{3x^{1+1}}{1+1} - \frac{7x^{0+1}}{0+1} + \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C$$

Formulas:

$(x^n)' = n \cdot x^{n-1}$ $\quad \int x^n dx = \frac{x^{n+1}}{n+1} + C$

$(e^x)' = e^x$ $\quad \int e^x dx = e^x + C$

$(\ln x)' = \frac{1}{x}$ $\quad \int \frac{1}{x} dx = \ln|x| + c$

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find concave up and concave down

$f(x) = e^{x^2-2x}$ $\quad f'g + g'f$

$f'(x) = e^{x2-2x} \cdot 2x-2$

$f''(x) = \underbrace{2}_{\text{derivative}} \cdot \underbrace{e^{x2-2x}}_{\text{copy}} + \underbrace{(2x-2)}_{\text{copy}} \underbrace{(2x-2)\, e^{x2-2x}}_{\text{derivative}}$

$= e^{x2-2x}\left[2 + (2x+2)^2\right]$

$= e^{x2-2x}\left[\cancel{4}\ 2 + 4x^2 - 8x + 4\right]$

$= e^{x2-2x}\left[4x^2 - 8x + 6\right] = 0$ Quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{4 \pm \sqrt{16-24}}{4}$$

$$\frac{4 \pm \sqrt{-8}}{4} = \text{no solution}$$

put 0 in any one $= +$

$(-\infty, \infty)$ concave up.

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Rules

$$\int f'(x) f^n(x)\, dx = \frac{f^{n+1}(x)}{n+1} + c$$

$$\int f'(x) e^{f(x)}\, dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int (3x^2+1) e^{x^3+x}\, dx$$

$$e^{x^3+x} + c$$

guess $f(x) = x^3 + x$
Check $f'(x) = f'(x) = 3x^2+1$

$$\int \frac{(3x^2+1)}{x^3+x}\, dx$$

guess $f(x) = x^3 + x$
Check $f'(x) = 3x^2+1$

$$\ln|x^3+x| + c$$

$$\int (3x^2+1)(x^3+x)^{21}\, dx$$

guess $f(x) = x^3 + x$
Check $f'(x) = 3x^2+1$

$$\frac{(x^3+x)^{22}}{22} + c$$

$$\int x e^{x^2+4}\, dx$$

guess $f(x) = x^2+4$
Check $f'(x) = 2x$

$$\frac{e^{x^2+4}}{2} + c$$

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extra = top.
missing = bottom.

$\int \frac{10x}{x^2+3}\,dx$ guess $f(x) = x^2+3$
check $f'(x) = 2x$

$5\ln|x^2+3| + c$

$\int \frac{x^2+1}{x}\,dx \rightarrow \frac{x^2}{x} + \frac{1}{x}\,dx$

$= x^1 + \frac{1}{x}\,dx \quad \frac{x^{1+1}}{1+1} + \ln|x| + c$

$\int \frac{x}{x^2+1}\,dx \rightarrow \frac{\ln|x^2+1|}{2} + c$ guess $f(x) = x^2+1$
check $f'(x) = 2x$

(11) $p'(x) = xe^{x^2}$ marginal price. derivative.
$p = ?$ price function integral
$p(1) = 5$

$p(x) = \int ex^{x^2}\,dx$ guess $f(x) = x^2$
check $f'(x) = 2x$

$p(x) = \frac{e^{x^2}}{2} + c$

$p(x)\ \frac{e^{1^2}}{2} + c = 5$

$1.35 + c = 5$

$c = 5 - 1.35 = c = 3.65$

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