76 %

Math 130

GUST#... .....Name... .....

Basic Calculus _Test Chapter 5

Time allowed 40 min.

1. (20 p.) Find

a)

$$\int \frac{4x^2-4}{4+x^3-3x}\,dx$$

$$\int \frac{4x^2-4}{4+u}\,dx$$

$u = x^3 - 3x$

$du = (3x^2 - 3)\,dx$ ✓

?? $du = 3(x^2-1)\,dx$ $\times 4$

$du = 12(4x^2-4)\,dx$

$dx = \frac{du}{12(4x^2-4)}$

$$\frac{1}{4}\int \frac{4x^2-4}{u}\cdot\frac{du}{12(4x^2-4)}$$

$$\frac{1}{4}\cdot\frac{1}{12}\int \frac{4x^2-4}{u}\cdot\frac{du}{(4x^2-4)}$$

$$\frac{1}{48}\int \frac{1}{u}\,du = \frac{1}{48}\ln u \xrightarrow{\text{translate Back!}}$$

$$= \boxed{\frac{1}{48}\ln(x^3-3x) + C}$$

why this ??

3.5 p.

b) $\int_0^1 (x^2+2x)^5(x+1)\,dx$

$u = x^2+2x$

$du = (2x+2)\,dx$

$du = 2(x+1)\,dx$

$dx = \frac{du}{2(x+1)}$

$\int u^5 \cdot (x+1)\,dx$

$\int u^5 \cdot (x+1) \cdot \frac{du}{2(x+1)}$

$\frac{1}{2}\int u^5\,du = \frac{1}{2}\cdot\frac{u^6}{6} = \frac{u^6}{12}$ → Trans Balik!

$= \frac{(x^2+2x)^6}{12} + C$

$\therefore \int_0^1 (x^2+2x)^5(x+1)\,dx = \frac{((1)^2+2\cdot(1))^6}{12} + C - \frac{((0)^2+2\cdot(0))^6}{12} + C$

$F(1) - F(0)$

$= 60.75 - 0 = 60.75$

5

c) $\int \frac{1-2x^5e^{x^3}}{x^3}\,dx$

$\int (1 - 2x^5 e^{x^3}) \cdot x^{-3}\,dx$

$\int (x^{-3} - 2x^2 e^{x^3})\,dx$

$= \frac{x^{-2}}{-2} - \frac{2x^3}{3}\cdot e^{x^3} + C$

$x^{-1} = \frac{1}{x} \rightarrow \ln x$

$\int \frac{1}{x^3} - \frac{2x^5 e^{x^3}}{x^3}\,dx$

$\int \frac{1}{x^3} - 2x^2 e^{x^3}\,dx$

$\int \frac{1}{x} - 2e^{x^3}\,dx$??

$= \ln x - 2e^{x^3} + C$

??

2.0 p.

$\sqrt[3]{x} = x^{\frac{1}{3}}$

d) $\int \left(\frac{1}{2}x^{-1} + \frac{1}{\sqrt[3]{x}} - 4\sqrt{x} + \ln 5\right) dx$

$\int \left(\frac{1}{2}x^{-1} + x^{-\frac{1}{3}} - 4 \cdot x^{\frac{1}{2}} + \ln 5\right) dx$

$= \frac{1}{2}\ln x + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - 4x^{\frac{3}{2}} + \ln 5\, x$

$= \frac{1}{2}\ln x + \frac{3x^{\frac{2}{3}}}{2} - \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + (\ln 5)x + C$ ✓ ✓ ✓ ✓

4.5 p.

2. (5 p.) Given marginal profit of producing x- units.

$$P'(x) = 100 - \frac{x}{50}$$

Find the profit function if the profit is 50 KD when the production is 10.

$P'(x) = 100 - \frac{1}{50}x$

$P(x) = 100x - \frac{1}{50} \cdot \frac{x^2}{2}$ ✓

$= 100x - \frac{x^2}{100} + C$

$P(50) = 100 \cdot (10) - \frac{(10)^2}{100} = 999 + C$

50 ✓

4 p. Find C $C = ?$