

Fall 2017 Exam 1

75/100

Name:

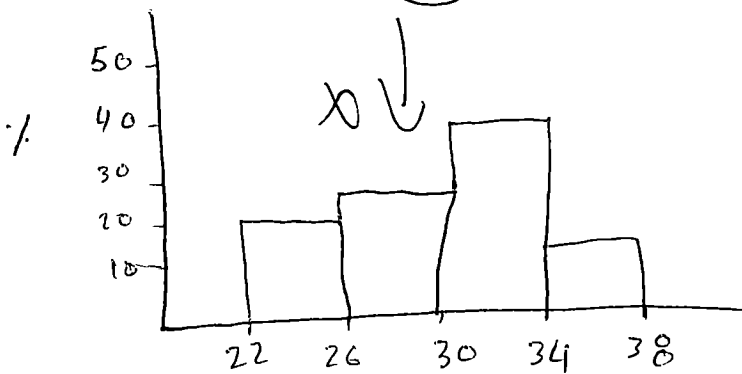
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Q1) (33pts) Ages of 40 employees in a company is summarized below.

Age	Frequency	Relative Frequency	Percentage	Cumulative Frequency
22 but less than 26	8	0.20	20%	8
26 but less than 30	10	0.25	25%	18
30 but less than 34	16	0.40	40%	34
34 but less than 38	6	0.15	15%	40
<b>Total</b>	40	1		

- a) Fill the table above.  
 b) Plot percentage polygon.

$$\frac{40 \times 5}{24} =$$



- c) Find the approximate mean of employees' ages.

$$\frac{(22 \times 8) + (26 \times 10) + (30 \times 16) + (34 \times 6)}{40} = 28$$

- d) Find the approximate standard deviation of employees' ages.

~~$$\frac{(22 \times 8)^2 + (26 \times 10)^2 + (30 \times 16)^2 + (34 \times 6)^2}{40} - (28)^2$$~~

~~$$25.84$$~~

~~$$5.98$$~~

~~$$18.75$$~~

Back Page (OK)

27) 
$$S = 15.5 \times \sqrt{\quad}$$

Q2) (42 pts) Exam scores of 12 students are given below as a stem-and-leaf display.

Stem	Leaf
7	0 4 8
8	0 4 6 8
9	0 2 4 6 8

70, 74, 78, 80, 84, 86, 88, 90, 94, 96, 98

- a) Find  
i. the quartile Q1

$$\checkmark \quad 78 \leftarrow \frac{n+1}{4}$$

- ii. the quartile Q2

$$\checkmark \quad \frac{n+1}{2} = 87$$

- iii. the quartile Q3

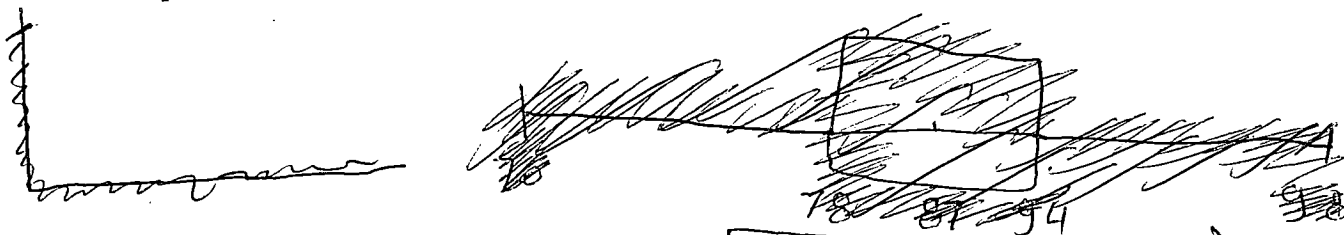
$$\checkmark \quad 3 \left( \frac{n+1}{4} \right) = 94$$

- iv. Range and IQR (the inter quartile range)

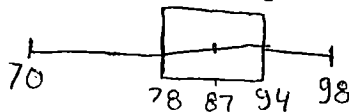
$$\text{range} = 98 - 70 = \boxed{28}$$

$$\text{IQR} = 94 - 78 = \boxed{16}$$

- b) Draw the Box-plot and comment on the distribution of the data.



- c) What percent of the grades is above 80?



skew to the right

$$\frac{8}{12} \times \frac{?}{100} = \frac{8 \times 100}{12} = 66.6\%$$

$$\frac{8}{12} \times 100 =$$

- d) If the mean of the data is 86 and the variance is 78.9, check whether 54 is an outlier or not. Explain your reason.

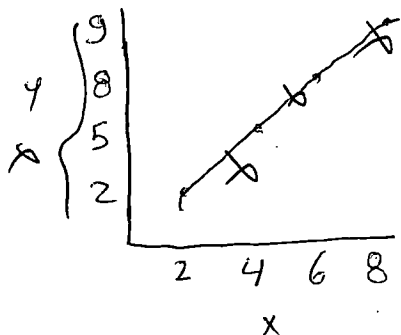
$$\frac{54 - 86}{\sqrt{78.9}} = -0.405 \quad \left( 36^2 \right)$$

~~No~~ it's not outlier because it's negative

Q3) (25 pts) Given the table,

X	Y
8	9
4	5
6	8
2	2

a) draw the scatter plot and comment on it.



There is a positive relation

$$\begin{array}{l} 20 \div 4 = 5 \\ 24 \div 4 = 6 \end{array}$$

b) find the covariance between X and Y and comment on your result.

$$Cov = \frac{(8-5)(9-6) + (4-5)(5-6) + (6-5)(8-6) + (2-5)(2-6)}{4}$$

$$= \frac{-60 + 72}{4}$$

The cov is positive

$$= 6.75$$

c) find the sample correlation coefficient between X and Y and comment on your result.

$$S_x = \frac{(8-5)^2 + (4-5)^2 + (6-5)^2 + (2-5)^2}{3} = 6.66$$

$$\sqrt{6.66} = 2.58$$

$$S_y = \frac{(9-6)^2 + (5-6)^2 + (8-6)^2 + (2-6)^2}{3} = 10$$

$$\sqrt{10} = 3.16$$

$$\frac{6.75}{2.58 \times 3.16} = 0.84$$

The correlation is positive  
strong (weak?)  
 $r \in [-1, 1]$

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**Chapter 2**

$$1) \text{ Width of interval} = \frac{\text{Range}}{\text{Number of Classes}}$$

$$2) \text{ Relative Frequency} = \frac{\text{Frequency of Each Class}}{\text{Total Number of Values}}$$

**Chapter 3**

$$1) \text{ Sample Mean } \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$2) \text{ Median is at } \frac{n+1}{2} \text{ ranked value}$$

$$3) \text{ Range} = x_{\text{Largest}} - x_{\text{Smallest}}$$

## 4) Sample Variance

$$\text{Var} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n-1}$$

## 5) Sample Standard Deviation

$$S = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n-1}}$$

## 6) Coefficient of Variation

$$CV = \frac{S}{\bar{x}} \times 100\%$$

## 7) Z-Score

$$Z = \frac{x - \bar{x}}{S}$$

## 8) Approximate Mean of a Frequency Distribution

$$\bar{x} = \frac{m_1 f_1 + m_2 f_2 + \dots + m_n f_n}{n}$$

$$\text{where } n = f_1 + f_2 + \dots + f_n$$

## 9) Approximate Standard Deviation of a Frequency Distribution

$$S = \sqrt{\frac{(m_1 - \bar{x})^2 f_1 + (m_2 - \bar{x})^2 f_2 + \dots + (m_n - \bar{x})^2 f_n}{n-1}}$$

10) First Quartile  $Q_1$  & Third Quartile  $Q_3$ 

$$Q_1 \text{ is at } \frac{n+1}{4} \text{ ranked value}$$

$$Q_2 \text{ is at } \frac{n+1}{2} \text{ ranked value (the median)}$$

$$Q_3 \text{ is at } 3 \left( \frac{n+1}{4} \right) \text{ ranked value}$$

## 11) Inter-quartile Range

$$IQR = Q_3 - Q_1$$

## 12) Covariance

$$\text{Cov}(X, Y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

## 13) Correlation Coefficient

$$r_{XY} = \frac{\text{Cov}(X, Y)}{S_X \cdot S_Y}$$

$S_X$  = standard deviation of X

$S_Y$  = standard deviation of Y

$$= \frac{(22-28)^2 \times 8 + (26-28)^2 \times 10 + (30-28)^2 \times 16 + (34-28)^2 \times 10}{39}$$

$$= \text{\textcircled{E}} 15.5$$