

Math 121  
Basic Stat

## Probability and Statistics - Test 2

Name:

GUST ID:

Thursday 16th April, 2015

Notes: For full credit, please show your calculation methods. Scientific calculators are not permitted, but non-scientific calculators are permitted.

### Questions:

1) For the data in the table:

X	P(X)
1	0.2
2	0.3
3	$a$
4	0.05
5	0.1
6	0.05

a) Find the value of  $a$

$$0.2 + 0.3 + a + 0.05 + 0.1 + 0.05 = 1$$

$$a = 0.3$$

b) Find the expected value of  $X$ .

$$(1 \cdot 0.2) + (2 \cdot 0.3) + (3 \cdot 0.3) + (4 \cdot 0.05) + (5 \cdot 0.1) + (6 \cdot 0.05) = 2.71$$

1

2) Khalid buys 10 chairs for his employees. The probability that a chair is faulty is 0.05. Assume a binomial distribution.

a) Find the probability that 2 chairs are faulty.

$$n = 10$$
$$p = 0.05$$
$$x = 2$$

$$\frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$
$$\frac{10!}{2!(10-2)!} \cdot 0.05^2 \cdot (1-0.05)^{10-2}$$
$$= \boxed{0.0746347285}$$

b) Find the probability that less than 2 chairs are faulty.

$$n = 10$$
$$p = 0.05$$
$$x = 0, 1$$

$$P(x < 2) = P(x=0) + P(x=1)$$

$$\left( \frac{10!}{0!(10-0)!} \cdot 0.05^0 \cdot (1-0.05)^{10-0} \right) + \left( \frac{10!}{1!(10-1)!} \cdot 0.05^1 \cdot (1-0.05)^{10-1} \right)$$
$$0.59873 + 0.3151 = \boxed{0.91386}$$

c) Find the probability that at least 1 chair is faulty.

$$P(x > 1) = 1 - P(x=0)$$
$$1 - \left( \frac{10!}{0!(10-0)!} \cdot 0.05^0 \cdot (1-0.05)^{10-0} \right)$$
$$= 1 - 0.59873 = \boxed{0.4012630608}$$

3) The number of phone calls arriving at a call center has a mean of 6 calls per hour. Assume a Poisson distribution.

a) What is the probability that there are no calls in a given hour?

$$\lambda = 6$$
$$x = 0$$

$$\frac{e^{-\lambda} \cdot \lambda^x}{x!} = \boxed{0.002478}$$

b) What is the probability that there are 6 calls in a given hour?

$$\lambda = 6$$
$$x = 6$$

$$\frac{e^{-6} \cdot 6^6}{6!} = \boxed{0.160623141}$$

c) What is the probability that there are 4 or 5 calls in a given hour?

$$\lambda = 6$$
$$x = 4 \text{ ; } 5$$

$$\frac{e^{-6} \cdot 6^4}{4!} + \frac{e^{-6} \cdot 6^5}{5!} = \boxed{0.0214998279}$$

4) A manager of a factory studies the distribution of the weights of bricks. He finds that the distribution is normal, with mean 5kg, and standard deviation 0.1kg.

a) What is the probability that a random brick is less than 4.85kg?

$$P(x < 4.85)$$
$$= P\left(z < \frac{4.85 - 5}{0.1}\right) = P(z < -1.5)$$
$$= \boxed{0.066811}$$

b) What is the probability that a random brick is between than 4.8kg and 5.1kg?

$$P(4.8 < x < 5.1)$$

$$P\left(\frac{4.8 - 5}{0.1} < z < \frac{5.1 - 5}{0.1}\right)$$

$$P(-2 < z < 1)$$

$$0.2275 < z < 0.84134$$

$$0.84134 - 0.2275 = \boxed{0.61384}$$

c) 10% of the bricks are lighter than  $x$ kg, and 90% are heavier than  $x$ kg. What is  $x$ ? (i.e. find  $x$  so that  $P(X < x) = 0.10$ ).

$$P(X < -2.32) = 0.10$$

$$\alpha = 0.10$$

$$z = -2.32$$

$$\alpha = 2.6 + M$$

$$\alpha = M + 2.6$$

$$\sigma = 0.1$$

$$\alpha = 2.6 + M$$

$$M = 5$$

$$x =$$

$$x = -2.32 \cdot 0.1 + 5 = 4.768$$

5) An airplane arrives at a random time between 5:20pm and 5:30pm. Assume that the time follows a uniform distribution.

a) Write the probability density function.

b) What is the probability that the arrival time is between 5:20pm and 5:24pm?

c) What is the probability that the arrival time is between 5:30pm and 5:35pm?

END OF QUESTIONS