Probability and Statistics - Test 2

Name:..

GUST ID:...

Dr. K. Dingle

Tuesday 25th April, 2017

33/37

Notes: For full credit, please show your calculation methods.
Questions:
1) For the data in the table:

Y	P(Y)
10	0.12
15	0.20
20	0.05
25	0.06
30	0.57

a) Find the expected value of Y.

$E(x) = n \cdot \pi$

$(10 \cdot 0.12) + (15 \cdot 0.2) + (20 \cdot 0.05) + (25 \cdot 0.06) + (30 \cdot 0.57)$

$= \boxed{23.81}$ ✓

b) Find the probability that Y is less than 25. $\boxed{0.06}$ ×

0.5/3 (Y<25)

2) Seven patients try a new drug to cure a disease. The probability that the drug cures a patient is 0.37. Assume a binomial distribution.
a) Find the probability that the drug cures 2 patients.

$n = 7$
$\pi = 0.37$
$x = 2$

$\frac{n!}{x!(n-x)!} \cdot \pi^x \cdot (1-\pi)^{n-x}$

$\frac{7!}{2!(7-2)!} \cdot 0.37^2 \cdot (1-0.37)^{7-2}$

$= 0.285315558171$

b) Find the probability that the drug cures more than 5 patients.

$n = 7$ ✓

$\pi = 0.37$ ✓

$x = P(x > 5) = 1 - P(x = 0)$ ✓ ?

$1 - \left(\frac{7!}{0!(7-0)!} \cdot 0.37^0 \cdot (1-0.37)^{7-0}\right) = 0.950477721$ ✗

1.5/3

3) On average, I receive 2.3 emails per hour while in my office. Assume a Possion distribution.

a) What is the probability that I receive 3 emails in an hour?

$\lambda = 2.3$

$x = 3$

$f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$= \frac{e^{-2.3} \cdot 2.3^3}{3!} = 0.203308225$

b) What is the probability that I receive less than 3 emails in two hours?

$\lambda = 2.3(2) = 4.6$

$x = P(x < 3) = P(x=0) + P(x=1) + P(x=2)$

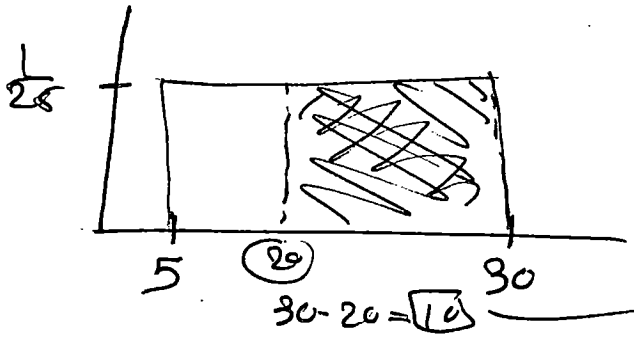
$\frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-4.6} \cdot 4.6^0}{0!} + \frac{e^{-4.6} \cdot 4.6^1}{1!} + \frac{e^{-4.6} \cdot 4.6^2}{2!} = 0.162638$

4) The time taken to read an email varies betwee 5 and 30 seconds. Assume that the time taken follows a uniform distribution.

a) Write the probability density function.

$$f(x) = \begin{cases} \frac{1}{30-5} = \frac{1}{25}, & 5 < x < 30 \\ 0, & \text{Other wise} \end{cases}$$

b) What is the probability that the time taken is between 20 and 35 seconds?



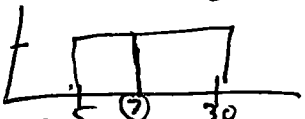
$20 < x < 35$

$= 20 < x < 30$

$\frac{1}{25} \cdot 10 = 0.4$

c) What is the probability that the time taken is exactly 7.5 seconds?

$P(X = 7.5) = 0$



$7 - 7 = 0$

$0 \cdot \frac{1}{25} = 0$

5) The afternoon temperature during a given week follows a normal distribution, with mean 31.0 degrees, and standard deviation 4.3 degrees.

a) What is the probability that on a random day the temperature is more than 33.5 degrees?

$\mu = 31.0$

$\sigma = 4.3$

$P(x > \frac{33.5 - 31.0}{4.3})$

$= P(z > 0.581) = 1 - 0.71904$

$= 0.28096$

b) What is the probability that on a random day the temperature is between 28 and 34 degrees?

$P(28 < x < 34)$

$P(\frac{28 - 31.0}{4.3} < z < \frac{34 - 31.0}{4.3})$

$= P(-0.64 < z < 0.64)$

$P(0.24510 < z < 0.75490)$

$= 0.75490 - 0.24510 = 0.5098$

c) What is the probability that on a random day the temperature is less than 50 degrees?

$P(x < 50)$

$= P(Z < \frac{50 - 31.0}{4.3})$

$= P(Z < 4.41860951)$ ✓

$\frac{2}{3}$ =?

d) On 16% of random days the temperature is less than x. What is x? (i.e. find x so that $P(X < x) = 0.16$).

$P(x < \alpha) = 0.16$

$-0.99 \rightarrow z$

$z = -0.99$

$\mu = 31.0$

$\sigma = 4.3$

$x = z \cdot \sigma + \mu$

$= -0.99 \cdot 4.3 + 31.0$

$= 26.743$ ✓

6) **BONUS** A binomial distribution has n trials, with $\pi = 0.4$. Given that $P(X = 3) = 0.2304$, find n.

$n = ?$

$\pi = 0.4$

$x = 3$

$\frac{1}{2}$

$\frac{n!}{3!(n-3)!} \cdot 0.4^3 \cdot (1-0.4)^{n-3} = 0.2304$

END OF QUESTIONS

$\frac{1}{3!(3)!} \cdot 0.4^3 \cdot (1-0.4)^{n-3} = 0.2304$

$1 \cdot 0.4^3 \cdot (1-0.4)^{n-3} = 0.2304$

$0.64 \cdot n-3 \ln(1-0.4) = \ln 0.2304$

$\frac{0.64 \cdot n-3}{0.64} = \frac{0.2304}{0.64}$

$n-3 = 0.6$

$n = 3.6$ ✗