Practice'booklet

Fall 2017

Math 121

Basic Probability and Statistics

Old Exams and Solutions

Dr. Harun Aydilek

Fall 2012

- Quizzes and Solutions

- Exam1 and Solutions

- Midterm exam and solutions

- Exam2 and Solutions

- Final exam and solutions

Summer 2012

- Quizzes and Solutions

- Exam1 and Solutions

- Exam2 and Solutions

- Additional practice problems and solutions

- Final exam and solutions

① Consider the data set to find

18, 31, 29, 30, 24, 27

a) Q_1, Q_2, Q_3

b) Draw   Box-plot

c) By using b) ; comment on the distribution
of the data

6

②

Price (x)	Sales (Y)
2	18
6	13
7	8

a) Find covariance between Price (x) and Sales (Y)

Find correlation coefficient

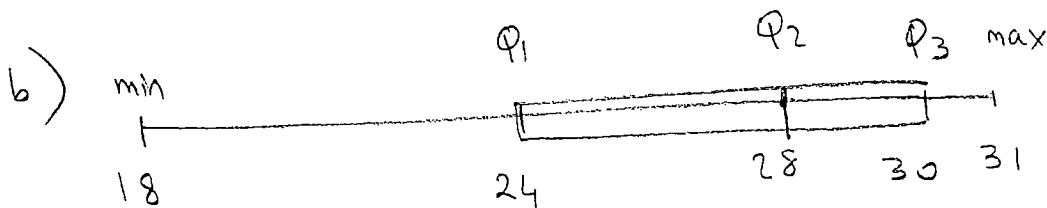Comment on the relation between Price $^{(x)}$ and Sales $^{(Y)}$

(3

① Sorted data : 18, 24, 27, 29, 30, 31 ⟹ n=6

a) Q_1 position $= \frac{n+1}{4} = \frac{7}{4} = 1.75$ ⟹ Q_1 position = 2

⟹ $Q_1 = 24$

Q_2 position $= 2\left(\frac{n+1}{4}\right) = 2\left(\frac{7}{4}\right) = 3.5$ ⟹ $Q_2 = \frac{3^{rd} \text{ data} + 4^{th} \text{ data}}{2}$

$= \frac{27 + 29}{2} = 28$

Q_3 position $= 3\left(\frac{n+1}{4}\right) = 3\left(\frac{7}{4}\right) = 5.25$ ⟹ Q_3 is the 5th data

⟹ $Q_3 = 30.$

b)

c) The data set is left skewed.

② a) $\bar{X} = \frac{2+6+7}{3} = 5$; $\bar{Y} = \frac{18+13+8}{3} = 13$

$cov(X,Y) = \frac{(2-5)(18-13) + (6-5)(13-13) + (7-5)(8-13)}{3-1} = \frac{-15 + 0 + (-10)}{2}$

$= \frac{-25}{2} = -12.5$

b) $S^2_x = \frac{(2-5)^2 + (6-5)^2 + (7-5)^2}{3-1} = \frac{9+1+4}{2} = 7$ ⟹ $S_x = \sqrt{7} = 2.65$

$S^2_Y = \frac{(18-13)^2 + (13-13)^2 + (8-13)^2}{3-1} = \frac{25+0+25}{2} = 25$ ⟹ $S_Y = \sqrt{25} = 5$

$corr(X,Y) = r = \frac{cov(X,Y)}{S_x S_Y} = -0.945$

c) There is very strong negative linear relation between Price & Sales.

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1) $P(A) = 0.35$, $P(B) = 0.40$ and A and B are Independent. Find $P(A \cup B)$.

2) $P(A) = 0.30$, $P(A|B) = 0.20$ and $P(B) = 0.60$. Find $P(A \cup B)$

3) $P(A) = 0.6$, $P(B') = 0.5$ and $P(A \cap B) = 0.30$ Are A and B Independent?

(5

1) $P(A) = 0.35$, $P(B) = 0.40$ and A and B are Independent. Find $P(A \cup B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ⟩ by independence property

$= 0.35 + 0.40 - P(A) \cdot P(B)$

$= 0.35 + 0.40 - 0.35 \times 0.40$

$= 0.61$

2) $P(A) = 0.30$, $P(A|B) = 0.20$ and $P(B) = 0.60$. Find $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.30 + 0.60 - 0.12$

$= 0.78$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$0.20 = \dfrac{P(A \cap B)}{0.60}$

$0.20 \times 0.60 = P(A \cap B)$

$0.12 = P(A \cap B)$

) $P(A) = 0.6$, $P(B') = 0.5$ and $P(A \cap B) = 0.30$ Are A and B independent?

$P(B) = 1 - 0.5 = 0.5$

$P(A \cap B) \overset{?}{=} P(A) P(B)$

$0.30 \overset{?}{=} 0.6 \times (1 - 0.5)$

$0.30 \overset{?}{=} 0.6 \times 0.5$

YES!

Summer II 2012 Quiz 3

Show all your work to get full credit.

Q. According to a survey, 35 % of the drivers are female. Last year, 20 % percent of female drivers had an accident and 30 % of male drivers had an accident.

  a) What is the probability that a randomly selected driver had an accident last year?

  b) If a randomly selected driver had an accident, what is the probability that the driver was male?
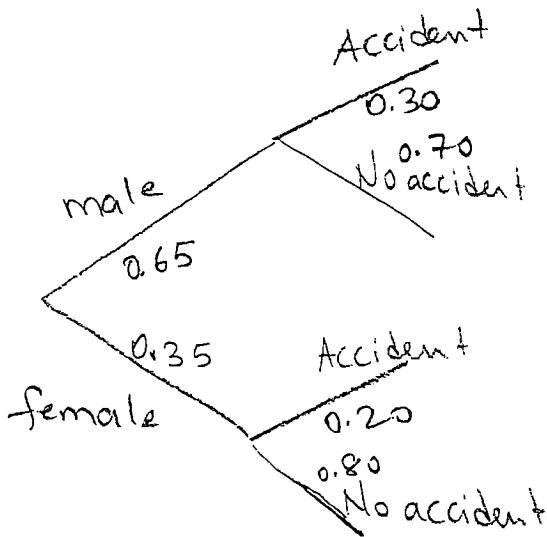
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Summer II 2012 Quiz 3

Solutions

Show all your work to get full credit.

Q. According to a survey, 35 % of the drivers are female. Last year, 20 % percent of female drivers had an accident and 30 % of male drivers had an accident.

a) What is the probability that a randomly selected driver had an accident last year?

$$P(Accident)$$

$$= P(male \cap Accident) + P(Female \cap Accident)$$

$$= 0.65 \times 0.30 + 0.35 \times 0.20$$

$$= 0.1950 + 0.070$$

$$= 0.265$$

b) If a randomly selected driver had an accident, what is the probability that the driver was male?

$$P(male \mid Accident) = \frac{P(male \cap Accident)}{P(Accident)}$$

$$= \frac{0.65 \times 0.30}{0.265} = \frac{0.195}{0.265} = 0.7358$$

Summer II 2012 Quiz 4

1) (20 points) Probability density function for X is given.

X	3	7	13
P(X)	0.3	0.5	0.2

    a) Find the expected value, E(X).

    b) Find the variance of X, Var(X).

2) (40 points) Customer service of a company receives 1.5 complaints per week on average. What is the probability that the customer service receives

    a) 2 complaints in one week?

    b) 5 complaints in four weeks?

3) (40 points) It is assumed that the probability of rain in winter is 0.3 for a random day. If the next 8 days are observed, what is the probability that

    a) it will rain for 2 days?

    b) it will rain for at most 6 days?

1

Summer II 2012 Quiz 4 Solutions

1) (20 points) Probability density function for X is given.

X	3	7	13
P(X)	0.3	0.5	0.2

a) Find the expected value, E(X).

$$E(X) = 3 \times 0.3 + 7 \times 0.5 + 13 \times 0.2 = 0.9 + 3.5 + 2.6 = 7$$

b) Find the variance of X, Var(X).

$$Var(X) = (3-7)^2 \times 0.3 + (7-7)^2 \times 0.5 + (13-7)^2 \times 0.2$$
$$= 4.8 + 0 + 7.2 = 12$$

2) (40 points) Customer service of a company receives 1.5 complaints per week on average. What is the probability that the customer service receives Poisson $\lambda = 1.5$ per week
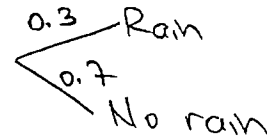
a) 2 complaints in one week?

$$P(X=2) = e^{-1.5} \frac{(1.5)^2}{2!} = 0.2510$$

b) 5 complaints in four weeks? $\longrightarrow$ new λ is $4 \times 1.5 = 6$

$$P(X=5) = e^{-6} \cdot \frac{6^5}{5!} = 0.1606$$

3) (40 points) It is assumed that the probability of rain in winter is 0.3 for a random day. If the next 8 days are observed, what is the probability that Binomial 0.3 Rain 0.7 No rain $n = 8$

a) it will rain for 2 days?

$$P(X=2) = {}_8C_2 (0.3)^2 (0.7)^6 = 0.2965$$

b) it will rain for at most 6 days?

$$P(X \le 6) = P(X=0) + P(X=1) + \cdots + P(X=6)$$
$$= 1 - \left\{ P(X=7) + P(X=8) \right\}$$
$$= 1 - \left\{ {}_8C_7 (0.3)^7 (0.7)^1 + {}_8C_8 (0.3)^8 (0.7)^0 \right\}$$
$$= 1 - \left\{ 0.0012 + 0.0001 \right\} = 1 - 0.0013$$
$$= 0.9987$$

1

Summer II 2012 Quiz 5

If Z is the standard normal random variable, answer the following questions

1) $P(Z \leq 1.76) = ?$

2) $P(1.53 \leq Z < 2.07) = ?$

3) $P(Z > -1.42) = ?$

4) $P(Z > a) = 0.9082$, then find a.

5) $P(-b \leq Z \leq b) = 0.8764$, then find b.

Summer II 2012 Quiz 5   Solutions

If Z is the standard normal random variable, answer the following questions

1) $P(Z \le 1.76) = ?$
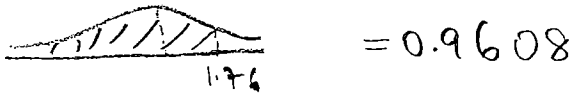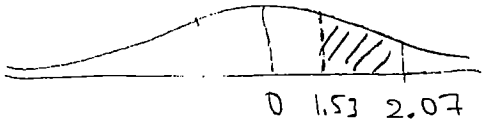

$= 0.9608$

2) $P(1.53 \le Z < 2.07) = ?$

$= 0.9808 - 0.9370 = 0.0438$

3) $P(Z > -1.42) = ?$

$= 1 - 0.0778 = 0.9222$

4) $P(Z > a) = 0.9082$, then find a.
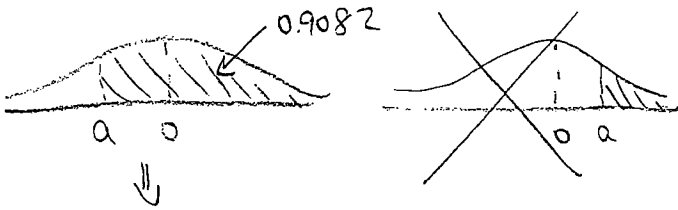
$= 1 - 0.9082 = 0.0918 \Rightarrow a = -1.33$

5) $P(-b \le Z \le b) = 0.8764$, then find b.
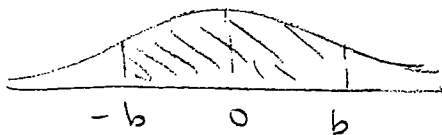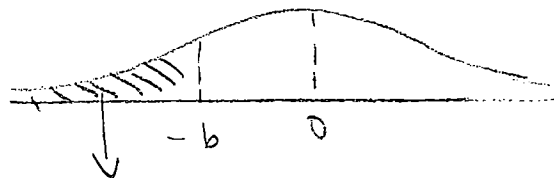
$$\frac{(1 - 0.8764)}{2} = \frac{0.1236}{2} = 0.0618$$

$$\Rightarrow -b = -1.54 \Rightarrow b = 1.54.$$

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Summer II 2012 Quiz 6

1. X is a normal random variable with a mean of 15 and standard deviation of 4.

a) $P(X > 12) = ?$

b) $P(10 < X < 20) = ?$

c) $P(X > n) = 0.1292$, then find n.

2. Temperature during a summer day in Kuwait is normally distributed with a mean of 46 degrees and the variance is 5. What is the probability that the temperature of a randomly selected day in summer will be at least 43 degrees?

$\left(13\right.$

Summer II 2012 Quiz 6 Solutions $\mu = 15, \sigma = 4$

1. X is a normal random variable with a mean of 15 and standard deviation of 4.

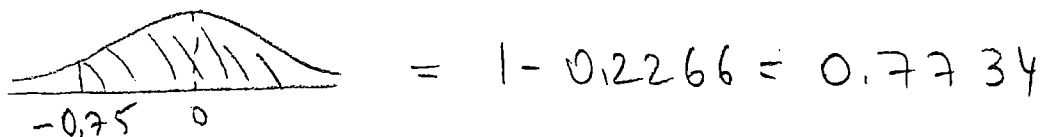a) $P(X > 12) = ?$ $P\left(\dfrac{X-\mu}{\sigma} > \dfrac{12-\mu}{\sigma}\right) = P\left(Z > \dfrac{12-15}{4}\right) = P(Z > -0.75)$

$= 1 - 0.2266 = 0.7734$

-0.75   0

b) $P(10 < X < 20) = ?$ $P\left(\dfrac{10-\mu}{\sigma} < \dfrac{X-\mu}{\sigma} < \dfrac{20-\mu}{\sigma}\right) = P\left(\dfrac{10-15}{4} < Z < \dfrac{20-15}{4}\right)$

$P(-1.25 < Z < 1.25)$

-1.25   0   1.25

$= 0.8944 - 0.1056 = 0.7888$

a

c) $P(X > n) = 0.1292$, then find n. $\Rightarrow P\left(\dfrac{X-\mu}{\sigma} > \dfrac{n-\mu}{\sigma}\right) = P\left(Z > \boxed{\dfrac{n-15}{4}}\right)$

$P(Z > a) = 0.1292$

a   0   0   a $\Rightarrow$ 0   a $= 1 - 0.1292$
                                                        $= 0.8708$

$\Rightarrow a = 1.13 \Rightarrow \dfrac{n-15}{4} = \dfrac{1.13}{1} \Rightarrow n - 15 = (4)(1.13)$

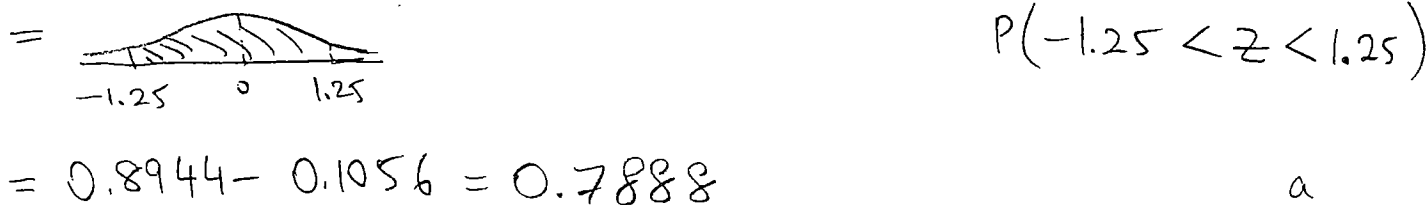$\Rightarrow n = 15 + (4)(1.13) = 15 + 4.52$
$= 19.52$

2. Temperature during a summer day in Kuwait is normally distributed with a mean of 46 degrees and the variance is 5. What is the probability that the temperature of a randomly selected day in summer will be at least 43 degrees? $= P(X > 43)$ $\sigma = \sqrt{5} = 2.24$

$= P\left(\dfrac{X-\mu}{\sigma} > \dfrac{43-\mu}{\sigma}\right) = P\left(Z > \dfrac{43-46}{\sqrt{5}}\right) = P\left(Z > \dfrac{-3}{2.24}\right)$
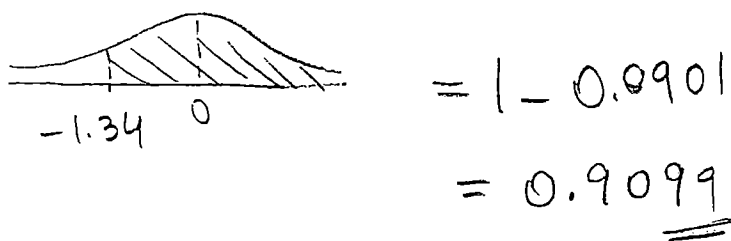
-1.34   0 $= 1 - 0.0901$ $P(Z > -1.34)$
                     $= 0.9099$

Math 121 Basic Probability and Statistics

Summer II 2012 Exam 1

Name :_____

ID # :_____

Show all your work to get full credit.

Question 1) (25 points) Exam scores for 8 students are given as 45, 38, 41, 52, 46, 32, 43, 39.

a) Find the quartiles Q1, Q2 and Q3.

$$32, 38, 39, 41, 43, 45, 46, 52$$

Q1 position $= \frac{n+1}{4} = \frac{32+1}{4} = 8.25 \Rightarrow 8^{rd}$ $\boxed{Q1 = 52}$

Q2 position $= 2\left(\frac{n+1}{4}\right) = 2\left(\frac{32+1}{4}\right) = 16.5$

Q1 position $= \frac{n+1}{4} = \frac{8+1}{4} = 2.25$ $\Rightarrow 2^{rd} \Rightarrow Q1 = 38$

Q2 position $= 2\left(\frac{n+1}{4}\right) = 2\left(\frac{9}{4}\right) = 4.5$ $Q2 \Rightarrow 5^{th}$ data 4th data $\frac{41+43}{2} = 42$

Q3 position $= 3\left(\frac{n+1}{4}\right) = 3\left(\frac{9}{4}\right) = 6.75$ $Q3 \Rightarrow 8^{th}$ $Q3 = 52$ 46
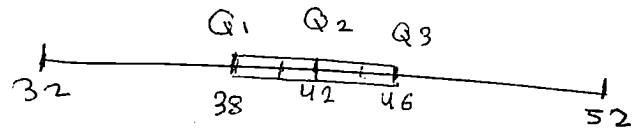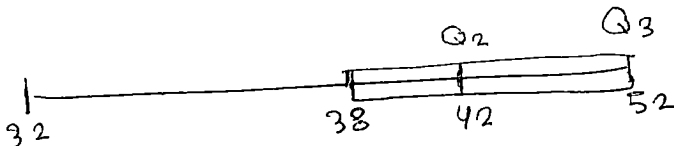
b) Draw the Box-Plot

c) What can you say about the distribution of the data? (i.e. Is it right skewed, symmetric or left skewed?)

Symmetric

d) Construct the stem-and-leaf display for the data.

```
3 | 2 8 9
4 | 1 3 5 6
5 | 2
```
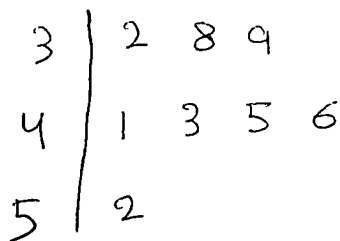
e) What percent of the data is less than 44

$$\frac{5}{8} \times 100 = 62.5\%$$

(15

Question 2) (25 points) Consider the following table for the parts below.

Class	Frequency
8-12	3
12-16	2
16-20	7
20-24	4

10
14 ~~16~~
18
22

16

a) Find the approximate mean.

$$\overline{x} = \frac{(10 \times 3) + (14 \times 2) + (18 \times 7) + (22 \times 4)}{16}$$

$$\overline{x} = \frac{30 + 28 + 126 + 88}{16} = 17$$

b) Find the approximate standard deviation

$$S^2 = \frac{3(10-17)^2 + 2(14-17)^2 + 7(18-17)^2 + 4(22-4)^2}{16 - 1}$$

$$S^2 = \frac{-21 + (-6) + 7 + 72}{15} = \frac{52}{15} = 3.466$$

$$S = \sqrt{S^2} = \sqrt{3.466} = 1.861$$

c) Plot the histogram for the frequency.

(16
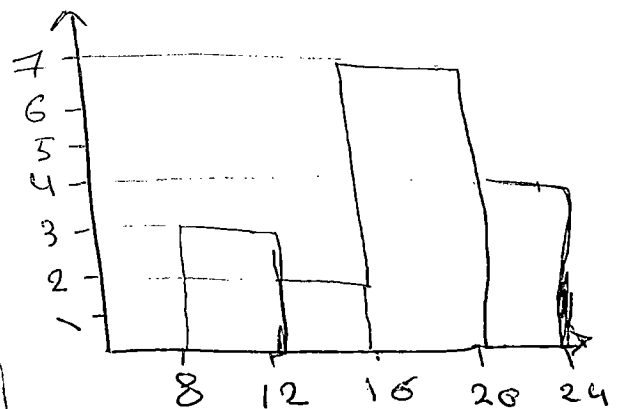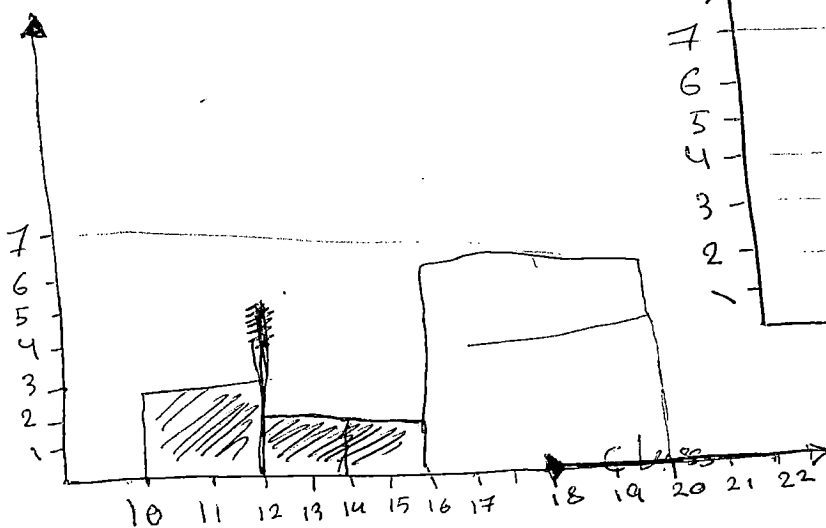
Question 3) (26 points) Education and hourly wage relation is given below.

Education (X)	Wage (Y)
3	6
7	12
8	15

a) Find the covariance between Education and Wage and comment on your result.

$$\bar{x} = \frac{3+7+8}{3} = \frac{18}{3} = 6$$

$$\bar{y} = \frac{6+12+15}{3} = \frac{33}{3} = 11$$

$$Cov(x,y) = \frac{(3-6)(6-11)+(7-6)(12-11)+(8-6)(15-11)}{3-1} = \frac{15+1+8}{2} = 12$$

$$Cov(x,y) = 12 \quad 7^{o}$$

There is positive linear relation between x and y

b) Find the correlation coefficient between Education and Wage and comment on your result.

$$Corr = \frac{12}{(1.626)(2)} = 2.460$$

(2.140)

$$S^2_x = \sqrt{\frac{(3-6)^2+(7-6)^2+(8-6)^2}{3-1}} = 2\sqrt{7} = 2.645$$

$$S^2_y = \sqrt{\frac{(6-11)^2+(12-11)^2+(15-11)^2}{3-1}} = \sqrt{21} = 4.582$$

$$xS = \sqrt{2.645} = 1.628$$

$$Sy = \sqrt{2} = 2.140$$

$$Corr = \frac{12}{(2.645)(4.582)} = 0.990$$

There is a strong positive linear relation between x and y

(17

Question 4) (24 points) Consider the table below for the following parts.

	White	Blue	Brown	
Toyota	55	40	15	110
Honda	25	35	10	70
Audi	35	20	5	60
	115	95	30	240

a) What is the probability that a randomly selected car is Honda?

$$\frac{70}{240} \approx 0.291$$

b) What is the probability that a randomly selected car is not white?

$$\frac{95 + 30}{240} \approx 0.5208$$

c) What is the probability that a randomly selected car is Toyota and the color is blue?

$$\frac{40}{240} \approx 0.1666$$

d) If randomly selected a brown car, what is the probability that it is Audi?

$$\frac{5}{30} \approx 0.1666$$

Math 121 Basic Probability and Statistics

Summer II 2012 Exam 1

Name : _____

ID # : _____

Show all your work to get full credit.

Question 1) (25 points) Exam scores for 8 students are given as 45, 38, 41, 52, 46, 32, 43, 39.

    a) Find the quartiles Q1, Q2 and Q3. $32, 38, 39, 41, 43, 45, 46, 52$

Q_1 position $= \frac{n+1}{4} = \frac{8+1}{4} = 2.25 \to 2 \Rightarrow Q_1$ is 2nd data $\Rightarrow Q_1 = 38$

Q_2 position $= 2\left(\frac{n+1}{4}\right) = 2\left(\frac{9}{4}\right) = 4.5 \to Q_2 = \frac{4^{th}\,data + 5^{th}\,data}{2} = \frac{41+43}{2} = 42$

Q_3 position $= 3\left(\frac{n+1}{4}\right) = 3\left(\frac{9}{4}\right) = 6.75 \to 7 \Rightarrow Q_3$ is 7th data

$$Q_3 = 46$$

    b) Draw the Box-Plot
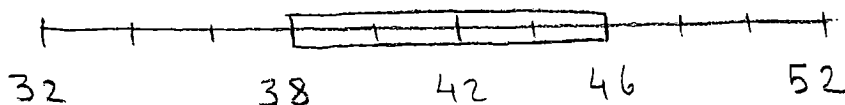
32   38   42   46   52

    c) What can you say about the distribution of the data? (i.e. Is it right skewed, symmetric or left skewed?)

    Symmetric.

    d) Construct the stem-and-leaf display for the data.

3	2 8 9
4	1 3 5 6
5	2

    e) What percent of the data is less than 44

$$\frac{5}{8} \times 100 = 62.5 \; \%$$

(19

Question 2) **(25 points)** Consider the following table for the parts below.

Class	Frequency	Mid Point
8-12	3	10
12-16	2	14
16-20	7	18
20-24	4	22
	16	

a) Find the approximate mean.

$$\bar{x} = \frac{3 \times 10 + 2 \times 14 + 7 \times 18 + 4 \times 22}{16} = \frac{30 + 28 + 126 + 88}{16}$$

$$= \frac{272}{16} = 17$$

b) Find the approximate standard deviation

$$\text{Variance} = S^2 = \frac{3 \times (10-17)^2 + 2 \times (14-17)^2 + 7 \times (18-17)^2 + 4 \times (22-17)^2}{16-1}$$
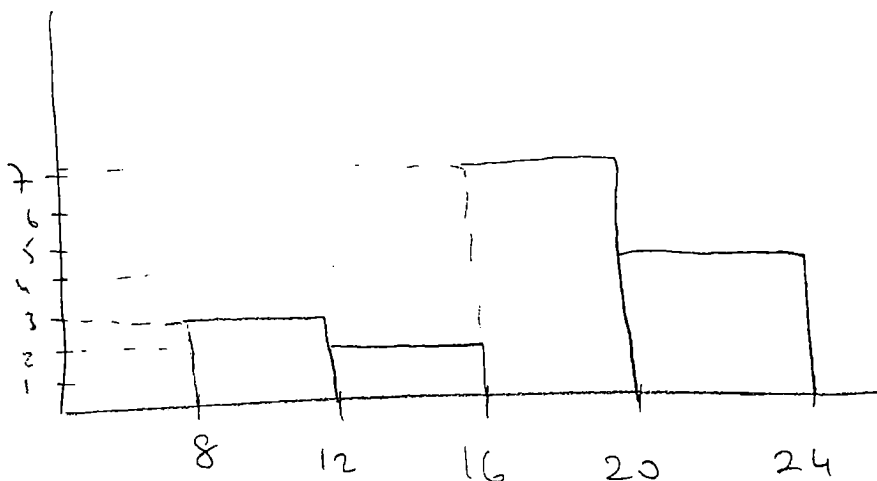
$$= \frac{147 + 18 + 7 + 100}{15} = \frac{272}{15} = 18.13$$

$$\text{Standard deviation} = S = \sqrt{\text{Variance}} = \sqrt{18.13} = 4.26$$

c) Plot the histogram for the frequency.

(20

Question 3) (25 points) Education and hourly wage relation is given below.

Education (X)	Wage (Y)
3	6
7	12
8	15

$$\overline{X} = \frac{3+7+8}{3} = \frac{18}{3} = 6$$

$$\overline{Y} = \frac{6+12+15}{3} = \frac{33}{3} = 11$$

a) Find the covariance between Education and Wage and comment on your result.

$$Cov(X,Y) = \frac{(3-6)(6-11) + (7-6)(12-11) + (8-6)(15-11)}{3-1}$$

$$= \frac{15 + 1 + 8}{2} = \frac{24}{2} = 12 > 0$$

$\Rightarrow$ There is positive linear relation between Education and Wage

b) Find the correlation coefficient between Education and Wage and comment on your result.

$$S_X^2 = \frac{(3-6)^2 + (7-6)^2 + (8-6)^2}{3-1} = \frac{9+1+4}{2} = \frac{14}{2} = 7$$

$$S_X = \sqrt{7} = 2.65$$

$$S_Y^2 = \frac{(6-11)^2 + (12-11)^2 + (15-11)^2}{3-1} = \frac{25+1+16}{2} = \frac{42}{2} = 21$$

$$S_Y = \sqrt{21} = 4.58$$

$$Corr(X,Y) = r = \frac{Cov(X,Y)}{S_X \cdot S_Y} = \frac{12}{(2.65)(4.58)} = 0.99$$

$\Rightarrow$ There is very strong & positive linear relation between Education and Wage

(21

Question 4) (24 points) Consider the table below for the following parts.

	White	Blue	Brown	Total
Toyota	55	40	15	110
Honda	25	35	10	70
Audi	35	20	5	60
Total	115	95	30	240

a) What is the probability that a randomly selected car is Honda?

$$\frac{70}{240} = 0.2917$$

b) What is the probability that a randomly selected car is not white?

$$\frac{95 + 30}{240} = \frac{125}{240} = 0.5208$$

c) What is the probability that a randomly selected car is Toyota and the color is blue?

$$\frac{40}{240} = 0.1667$$

d) If randomly selected a brown car, what is the probability that it is Audi?

$$\frac{5}{30} = 0.1667$$

$\left(22 \right.$

Math 121 Basic Probability and Statistics

Summer II 2012 Exam 2

Name :_____

ID # :_____

Show all your work to get full credit.

Question 1) (20 points)
It is given that 55 % of the students at GUST are female students. 40 % of female students register for a summer class and 35 % of male students register for a summer class. If we randomly select a student,

    a) what is the probability that s/he registers for a summer class?

    b) what is the probability the student is male if the student registered for a summer class?

Question 2) (10 points) In order to register for Fall semester, you need to select 2 electives out of 6 and 3 major courses out of 7. In how many different ways you can select your total of 5 courses in order to register for Fall semester?

23

Question 5) (20 points)

According to the past data, the average number of patients visiting the emergency service is β in one hour.

a) What is the probability that no patients visit in one hour?

Possien

$\lambda = 3$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^0 e^{-3}}{0!} = 0.04978$$

b) What the probability that 4 patients visit in two hours?   new $\lambda = 3 \times 2 = 6$

$$P(x = 4)$$

$$P(4) = \frac{6^4 e^{-6}}{4!} = 0.1338$$

c) What is the probability that at least two patients visit in one hour

$$P(x \geq 2)$$

$$= 1 - \left\{ P(x=1) + P(x=0) \right\}$$

$$= 1 - \left\{ \frac{3^1 e^{-3}}{1!} + \frac{3^0 e^{-3}}{0!} \right\} =$$

$$1 - \left\{ 0.1493 + 0.0497 \right\} = 0.801$$

Question 6) (15 points)

A farmer finds out that the weight of a tomato is uniformly distributed between 200 and 260 grams.

a) Write the probability density function and graph it.

b) What is the probability that a randomly selected tomato is less than 215 grams?

c) What is the expected weight of 10 tomatoes?

(25

Math 121 Basic Probability and Statistics

## Summer II 2012 Exam 2

Solutions

Name :_____

ID # :_____

Show all your work to get full credit.

Question 1) (20 points)
It is given that 55 % of the students at GUST are female students. 40 % of female students register for a summer class and 35 % of male students register for a summer class. If we randomly select a student,

a) what is the probability that s/he registers for a summer class?

Register 0.35
male — No 0.65
0.45
Register 0.40
female —
0.55 No 0.60
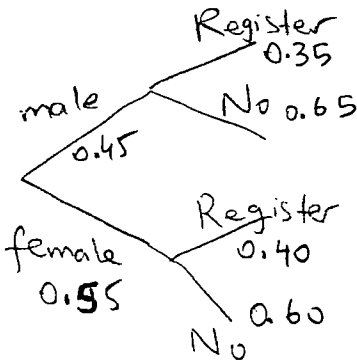
$P(\text{Register}) = 0.45 \times 0.35 + 0.55 \times 0.40$

$= 0.3775$

b) what is the probability the student is male if the student registered for a summer class?

$P(\text{Male}|\text{Register}) = \dfrac{P(\text{Male} \cap \text{Register})}{P(\text{Register})} = \dfrac{0.45 \times 0.35}{0.3775}$

$= 0.4172$

Question 2) (10 points) In order to register for Fall semester, you need to select 2 electives out of 6 and 3 major courses out of 7. In how many different ways you can select your total of 5 courses in order to register for Fall semester?

$_6C_2 \times _7C_3 = 15 \times 35 = 525$

(26

Question 3) (15 points) The probability density function for X is given below. X represents the number of classes taken by a student at GUST.

a) Find the missing value in the table and find the expected value of X, E(X).

X	3	4	5	6
P(X)	0.3	0.4	0.1	0.2

$E(x) = 3 \times 0.3 + 4 \times 0.4 + 5 \times 0.1$
$+ 6 \times 0.2 = 4.2$

$1 - (0.3 + 0.4 + 0.1)$

b) If GUST has 3500 students, what is the expected number of classes taken by GUST students?

$4.2 \times 3500 = 14700$

c) What is the probability that a randomly selected student will register for more than 4 classes?

$P(x > 4) = P(x = 5) + P(x = 6)$
$= 0.1 + 0.2 = \boxed{0.3}$

Question 4) (20 points) Historically, the probability of graduating from high school on time is 0.80. Out of 9 randomly selected students, Binomial $n = 9$ $0.80 \to$ on time
$0.20 \to$ not on time
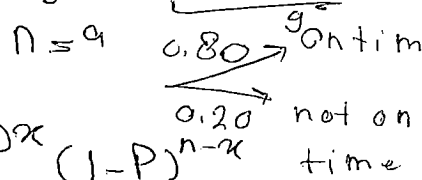
a) what is the probability that 5 graduate on time?

$P(x = 5)$ $\,_9C_5 \, (0.8)^5 (0.2)^{5-9}$ $P(x) = \,_nC_x \, p^x (1-p)^{n-x}$
$= 0.0661$

b) what is the probability that at most 7 graduate on time?

$P(x \le 7) = P(x = 0) + \cdots + P(x = 7)$
$1 - \{ P(x = 8) + P(x = 9) \}$
$= 1 - \{ \,_9C_8 \, (0.8)^8 (0.2)^1 + \,_9C_9 \, (0.8)^9 (0.2)^0 \}$
$1 - \{ 0.3020 + 0.13421 \} = \cancel{0.43562}$
0.5638

c) Find the mean and the variance for the graduates on time.

$E(x) = n \cdot p = 9 \times 0.80 = 7.2$

$Var(x) = n \cdot p(1-p) = 7.2 \times 0.2 = 1.44$

(24

Question 3) (15 points) The probability density function for X is given below. X represents the number of classes taken by a student at GUST.

a) Find the missing value in the table and find the expected value of X, E(X).

X	3	4	5	6
P(X)	0.3	0.4	0.1	

$$E(X) = 3 \times 0.3 + 4 \times 0.4 + 5 \times 0.1 + 6 \times 0.2$$
$$= 0.9 + 1.6 + 0.5 + 1.2 = 4.2$$

$$1-(0.3+0.4+0.1)$$
$$= 0.2$$

b) If GUST has 3500 students, what is the expected number of classes taken by GUST students?

$$= 4.2 \times 3500 = 14700$$

c) What is the probability that a randomly selected student will register for more than 4 classes?

$$P(X>4) = P(X=5) + P(X=6)$$
$$= 0.1 + 0.2 = 0.3$$

Question 4) (20 points) Historically, the probability of graduating from high school on time is 0.80. Out of 9 randomly selected students, Binomial. $n = 9$ $0.8 \quad G$ $0.2 \quad N$
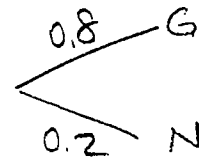
a) what is the probability that 5 graduate on time?

$$P(X=5) = {}_9C_5 (0.8)^5 (0.2)^4$$
$$= 0.0661$$

b) what is the probability that at most 7 graduate on time?

$$P(X \leq 7) = P(X=0) + \cdots + P(X=7)$$
$$= 1 - \{P(X=8) + P(X=9)\} = 1 - \{{}_9C_8 (0.8)^8 (0.2)^1 + {}_9C_9 (0.8)^9 (0.2)^0\}$$
$$= 1 - \{0.3020 + 0.1342\} = 1 - 0.4362$$
$$= 0.5638$$

c) Find the mean and the variance for the graduates on time.

$$E(X) = np = 9 \times 0.8 = 7.2$$

$$Var(X) = np(1-p) = 9 \times 0.8 \times 0.2 = 1.44$$

(27

Question 5) (20 points)

According to the past data, the average number of patients visiting the emergency service is 3 in one hour.

Poisson ; $\lambda = 3$

a) What is the probability that no patients visit in one hour?

$$P(X=0) = e^{-3} \cdot \frac{3^0}{0!} = 0.0498$$

b) What the probability that 4 patients visit in two hours? $\longrightarrow$ new $\lambda = 2 \times 3 = 6$

$$P(X=4) = e^{-6} \frac{6^4}{4!} = 0.1339$$

c) What is the probability that at least two patients visit in one hour $\longrightarrow \lambda = 3$

$$P(X \geq 2) = P(X=2) + P(X=3) + \ldots + P(X=\infty)$$

$$= 1 - \{P(X=0) + P(X=1)\} = 1 - \{0.0498 + 0.1494\}$$

$$= 1 - \{0.1992\} = 0.8008$$

$\nwarrow e^{-3} \cdot \frac{3^0}{0!}$ $\nwarrow e^{-3} \cdot \frac{3^1}{1!}$

Question 6) (15 points)

A farmer finds out that the weight of a tomato is uniformly distributed between 200 and 260 grams.

a) Write the probability density function and graph it.

$$f(x) = \begin{cases} \dfrac{1}{60} & 200 \leq X \leq 260 \\ 0 & otherwise \end{cases}$$
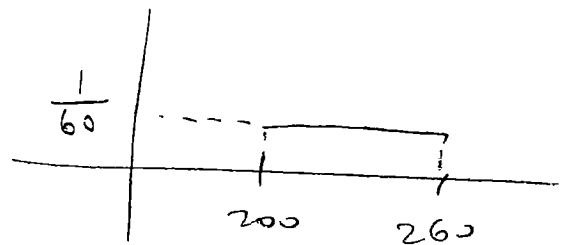
b) What is the probability that a randomly selected tomato is less than 215 grams?

$$P(X < 215) = 15 \times \frac{1}{60}$$
$$= 0.25$$
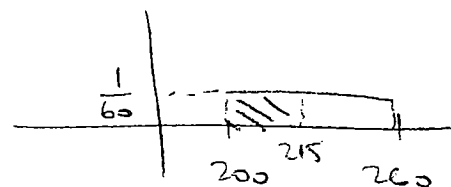
c) What is the expected weight of 10 tomatoes?

$$E(x) = \frac{200 + 260}{2} = 230$$

10 tomatoes $\longrightarrow$ $10 \times 230 = 2300 \, gr.$

Example: (Ch4)

	Econ	Math	Biology
Freshman	30	25	5
Sophomore	40	35	50
Junior	20	45	15

a) How many students are taking Math

b) what percent of Econ students are Fresmen?

c) If you randomly select a student what is the probability that the student takes Biology

d) If you randomly select a Junior, what is the probability that the student takes Math?

e) If you randomly select a Biology student, what is the probability that the student is a Sophomore?

f) If you randomly select a student, what is the probability that the student takes Math and the student is a Freshman

(29

Example: (Ch4) @GUST.IKWB MyGUST.com Solutions

	Econ	Math	Biology	
Freshman	30	25	5	60
Sophomore	40	35	50	125
Junior	20	45	15	80
	90	105	70	265

a) How many students are taking Math

105

b) What percent of Econ students are Fresmen?

$$\frac{30}{90} \times 100 = 33.3 \%$$

c) If you randomly select a student what is the probability that the student takes Biology

$$\frac{70}{265}$$

d) If you randomly select a Junior, what is the probability that the student takes Math?

$$\frac{45}{80}$$

e) If you randomly select a Biology student, what is the probability that the student is a Sophomore?

$$\frac{50}{70}$$

f) If you randomly select a student, what is the probability that the student takes Math and the student is a Freshman $= \frac{25}{265}$

(30

1) Consider the following probability density function for X.

X	2	4	10
P(X)	0.2	0.4	0.4

a) Find the expected value of X; E(X).

b) Find the variance of X, Var(X).

2) Assume that the probability of passing Math 121 for any student is 0.8. If we randomly select 6 students, what is the probability that

$$n = 6 \quad 0.8 \nearrow P$$
$$\searrow 0.2 \quad \text{not } P$$

a) only 5 of them will pass the course?

$$P(x=5) = 6C_5 \times (0.8)^5 \times (0.2)^{6-5} = 0.3932$$
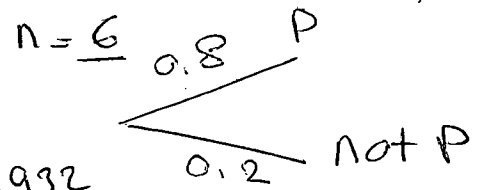
b) at most four of them will pass the course?

$$P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + \cdots P(x=4)$$
$$= 1 - \{ P(x=5) + P(x=6) \}$$
$$= 1 - \{ 6C_5 * 0.8^5 * 0.2^{6-5} + 6C_6 * 0.8^6 * 0.2^{6-6} \}$$

3) Assume that on average 3 earthquakes happen in Japan everyday. What is the probability that

a) 4 earthquakes happen in one day? $\lambda = 3$

$$P(x=4) = \frac{3^4 \cdot e^{-3}}{4!}$$
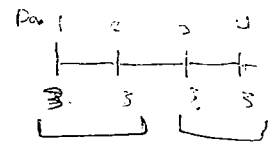
b) 7 earthquakes happen in two days? new $\lambda = 3 * 2 = 6$

$$P(x=7) = \frac{6^7 \cdot e^{-6}}{7!}$$

more than 2 earthquakes in Three days? $\lambda = 3*3 = 9$

$$P(x > 2) = P(3) + P(4) + \cdots$$
$$1 - \{ P(x=0) + P(x=1) + P(x=2) \}$$ (31

1) Consider the following probability density function for X.

X	2	4	10
P(X)	0.2	0.4	0.4

a) Find the expected value of X; E(X).

$$E(X) = 2 \times 0.2 + 4 \times 0.4 + 10 \times 0.4 = 0.4 + 1.6 + 4 = 6$$

b) Find the variance of X, Var(X).

$$Var(X) = (2-6)^2 \times 0.2 + (4-6)^2 \times 0.4 + (10-6)^2 \times 0.4$$
$$= 3.2 + 1.6 + 6.4 = 11.2$$

2) Assume that the probability of passing Math 121 for any student is 0.8. If we randomly select 6 students, what is the probability that (Binomial $n = 6$ pass 0.8, fail 0.2)
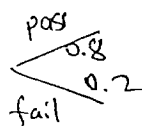
a) only 5 of them will pass the course?

$$P(X=5) = {}_6C_5 (0.8)^5 (0.2)^1$$

$$= 0.3932$$

b) at most four of them will pass the course?

$$P(X \leq 4) = P(X=0) + \cdots + P(X=4) = 1 - \{ P(X=5) + P(X=6) \}$$

$$= 1 - \{ {}_6C_5 (0.8)^5 (0.2)^1 + {}_6C_6 (0.8)^6 (0.2)^0 \} = 1 - \{ 0.3932 + 0.2624 \}$$

$$= 1 - 0.6553 = 0.3447$$

3) Assume that on average 3 earthquakes happen in Japan everyday. What is the probability that

a) 4 earthquakes happen in one day? (Poisson) $\lambda = 3/day$.

$$P(X=4) = e^{-3} \cdot \frac{3^4}{4!} = 0.1680$$

b) 7 earthquakes happen in two days? $\rightarrow$ new $\lambda = 2 \times 3 = 6$

$$P(X=7) = e^{-6} \cdot \frac{6^7}{7!} = 0.1377$$

(32

1. The service manager for a new automobile dealership reviewed dealership records of the past 20 sales of new cars to determine the number of warranty repairs he will be called on to perform in the next 90 days. Corporate reports indicate that the probability any one of their new cars needs a warranty repair in the first 90 days is 0.05. The manager assumes that calls for warranty repair are independent of one another and is interested in predicting the number of warranty repairs he will be called on to perform in the next 90 days for this batch of 20 new cars sold.

    a) Type of probability distribution?

    b) Probability of at least two warranty repairs?

2. The quality control manager of Marilyn's Cookies is inspecting a batch of chocolate chip cookies. When the production process is in control, the average number of chocolate chip parts per cookie is 6.0. The manager is interested in analyzing the probability that any particular cookie being inspected has more than 2 chip parts.

    a) Type of probability distribution?

    b) If a box contains 4 cookies, what is the probability that the there will be 20 chocolate chip parts in the box?

    c) Mean and variance of the number of chocolate chip parts per cookie?

3. A professor receives, on average, 2.5 e-mails from students per day. Poisson

$P(x=x) =$
$\dfrac{\lambda^x e^{-\lambda}}{x!}$

a) To compute the probability of receiving at least 3 e-mails on a random day, he will use what type of probability distribution?

$P(x \geq 3) = P(x=3) + P(x=4) + \ldots P(x=\infty)$ Poisson $\lambda = 2.5$

b) Find the probability in part a).

$P(x \geq 3) = 1 - \{ P(x=0) + P(x=1) + P(x=2) \}$

$1 - \left(\dfrac{0.2^0 e^{-0.2}}{0!} + \dfrac{0.2^1 e^{-0.2}}{1!} + \dfrac{0.2^2 e^{-0.2}}{2!} \right) = 1.14 \times 10^{-8}$

0.9014

c) What is the probability of receiving 7 emails during the weekend?

$P(x=7)$ 2 days   new $\lambda = 2 \times 2.5 = 5$

$P(7) = \dfrac{5^7 e^{-5}}{7!} = 0.1044$

4. A company has 125 personal computers. The probability that any one of them will require repair on a given day is 0.04.

a) To find the probability that exactly 20 of the computers will require repair on a given day, one will use what type of probability distribution?

Binomial. $n = 125$ 0.04 requir

                                          0.96 No requir

b) What is the probability in part a) ?

$P(x=20) = {}_{125}C_{20} (0.04)^{20} (0.96)^{125-20=105}$

$= 1.08 \times 10^{-7}$

c) What is the expected number of computers that require repair per day?

$E(x) = n \cdot p = 125 \times 0.04 = 5$

d) What is the expected number of computers that require repair in 5 days?

$5 \times 5 = \boxed{25}$ requir in 5 days

e) Assume that on average, one repair takes 15 minutes and costs 30 KD. Find the expected time needed and cost for repairs in 5 days

$\lambda = 5$

$E(repair) = 15 \times 25 = 375.$

$E(cost) = 30 \times 25 = 750.$

1. The service manager for a new automobile dealership reviewed dealership records of the past 20 sales of new cars to determine the number of warranty repairs he will be called on to perform in the next 90 days. Corporate reports indicate that the probability any one of their new cars needs a warranty repair in the first 90 days is 0.05. The manager assumes that calls for warranty repair are independent of one another and is interested in predicting the number of warranty repairs he will be called on to perform in the next 90 days for this batch of 20 new cars sold.

   a) Type of probability distribution?

   Binomial $n = 20$

   Repair
   0.05
   0.95
   No repair

   b) Probability of at least two warranty repairs?

   $$P(X \geq 2) = P(X=2) + \ldots + P(X=20) = 1 - \{ P(X=0) + P(X=1) \}$$

   $$= 1 - \{ P(X=0) + P(X=1) \} = 1 - \left\{ {}_{20}C_0 (0.05)^0 (0.95)^{20} + {}_{20}C_1 (0.05)^1 (0.95)^{19} \right\}$$

   $$= 1 - \{ 0.3585 + 0.3774 \} =$$

   $$= 0.2641$$

2. The quality control manager of Marilyn's Cookies is inspecting a batch of chocolate chip cookies. When the production process is in control, the average number of chocolate chip parts per cookie is 6.0. The manager is interested in analyzing the probability that any particular cookie being inspected has more than 2 chip parts.

   a) Type of probability distribution?

   Poisson . $\lambda = 6$ per cookie.

   $P(X > 2)$ ← one cookie ,

   b) If a box contains 4 cookies, what is the probability that the there will be 20 chocolate chip parts in the box?

   New $\lambda = 6 \times 4 = 24$.

   $\lambda = 6$

   $$P(X=20) = e^{-24} \cdot \frac{24^{20}}{20!} =$$

   $$= 0.0624$$

   $$= P(X=3) + P(X=4) + \ldots$$

   $$= 1 - \{ P(X=0) + P(X=1) + P(X=2) \}$$

   $$= \ldots$$

   $$= 0.9380$$

   c) Mean and variance of the number of chocolate chip parts per cookie?

   Mean $= \lambda = 6$ per cookie

   Variance $= \lambda = 6$ " "

(35

3. A professor receives, on average, 2.5 e-mails from students per day. Poisson $\lambda = 2.5$ /day

a) To compute the probability of receiving at least 3 e-mails on a random day, he will use what type of probability distribution?

Poisson. $\longrightarrow \lambda = 2.5$

b) Find the probability in part a).

$P(x \geq 3) = P(X=3) + P(x=4) + \cdots + P(x=\infty)$

$= 1 - \{ P(x=0) + P(x=1) + P(x=2) \} = 1 - \{ 0.0821 + 0.2052 + 0.2565 \}$

$= 0.4562$

c) What is the probability of receiving 7 emails during the weekend? 2 days $\Rightarrow$ new $\lambda = 2 \times 2.5 = 5$.

$P(X=7) = \dfrac{e^{-5} \cdot 5^7}{7!} = 0.1044$

4. A company has 125 personal computers. The probability that any one of them will require repair on a given day is 0.04.

a) To find the probability that exactly 20 of the computers will require repair on a given day, one will use what type of probability distribution? Binomial. $n = 125$

0.04 Repair
0.96 No repair.
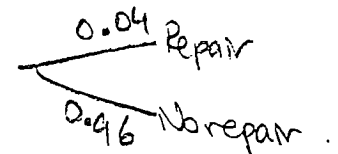
b) What is the probability in part a) ?

$P(X=20) = C_{125\ 20} (0.04)^{20} (0.96)^{105} = 0.0000 = \underline{\underline{0}}$

c) What is the expected number of computers that require repair per day?

$E(X) = n \cdot p = 125 \times 0.04 = \underline{\underline{5}}$

d) What is the expected number of computers that require repair in 5 days?

$5 \times 5 = 25$ repairs in 5 days

$\downarrow$ in one day
$\qquad$ days

e) Assume that on average, one repair takes 15 minutes and costs 30 KD. Find the expected time needed and cost for repairs in 5 days

$E(Time) = 15 \text{ min} \times 25 \text{ repairs} =$

$E(Cost) = 30 \times 25 = 750.$

(36

① Z is a standard normal random variable. Find

a) $P(Z \le 2.13) =$

b) $P(0 \le Z \le 1.72) =$

c) $P(Z = 1.38) =$

d) $P(Z > 2.75) =$

e) $P(1.16 < Z \le 2.13) =$

f) $P(Z \le -1.42) =$

g) $P(Z \ge -1.42) =$

h) $P(-1.42 \le Z \le 2.61) =$

i) $P(-1.42 \le Z \le 4.17) =$

j) $P(-3.96 \le Z \le 2.13) =$

② a) $P(Z \le a) = 0.9573 \Rightarrow a = ?$

b) $P(Z \le b) = 0.1469 \Rightarrow b = ?$

c) $P(Z \ge c) = 0.0694 \Rightarrow c = ?$

d) $P(Z \ge d) = 0.8340 \Rightarrow d = ?$

③ X has a normal distribution with mean 10 and variance 25.

a) $P(X < 18) = ?$

b) $P(6 < X \le 23) = ?$

c) $P(X = 17) = ?$

d) $P(X < k) = 0.1251 \Rightarrow k = ?$

e) $P(X > \ell) = 0.0418 \Rightarrow \ell = ?$

(37

1. a) 0.9834

   b) $0.9573 - 0.5 = 0.4573$

   c) 0

   d) $1 - 0.9970 = 0.0030$

   e) $0.9834 - 0.8770 = 0.1064$

   f) 0.0778

   g) $1 - 0.0778 = 0.9222$

   h) $0.9955 - 0.0778 = 0.9177$

   i) $= 1 - 0.0778 = 0.9222$ (OR it is ok to write $0.9990 - 0.0778 = 0.9212$)

   j) $= 0.9834 - 0 = 0.9834$ (OR it is ok to write $0.9834 - 0.001 = 0.9824$)

2. a) $a = 1.72$

   b) $b = -1.05$

   c) $c = 1.48$ (not -1.48)

   d) $d = -0.97$

3. a) $= P(z < 1.6) = 0.9452$

   b) $P(-0.8 < z \leq 2.6) = 0.9953 - 0.2119 =$

   c) 0

   d) $P\left(z < \dfrac{k-10}{5}\right) = 0.1251 \Rightarrow \dfrac{k-10}{5} = -1.15$

   $\Rightarrow k = (5)(-1.15) + 10 = 4.25$

   e) $P\left(z > \dfrac{l-10}{5}\right) = 0.0418 \Rightarrow \dfrac{l-10}{5} = 1.73$

   $\Rightarrow l = 10 + (5)(1.73) = 18.65$

(38

Gulf University for Science and Technology

Basic Probability and Statistics

Final Examination – Summer II 2012

Course Code: MATH 121   **Section:.......**

Instructor: Dr. Harun Aydilek   Date: September 6, 2012

Student Name: _____

Student Number: _____

Aids Allowed You can use a calculator, but it cannot be shared.

Instructions:

1. **The exam is double sided.**
2. **You can use the formula sheets and tables given to you.**

This examination has a cover page, 10 pages for questions, and one blank page at the end for extra space. Total pages is 12 counting double side. In addition, formula sheets and tables are 5 pages in total. Before you start the examination please verify them.

No Questions are allowed during the examination

Student signature: _____

(39

1. Based on the scores of 40 students, the following table is constructed.

a. (4 points) Fill the following table.

Class	Frequency	Cumulative Frequency	Relative Frequency	Percentage
0 but less than 6	8	8		
6 but less than 12		12		
12 but less than 18		22		
18 but less than 24		28		
24 but less than 30		40		
Total				

b. (2 points) What percent of the students scored less than 18?

c. (3 points) Plot the histogram for the percentage.

2

2. Exam 1 scores of 14 students are given below as a stem-and-leaf display.

Stem	Leaves
5	2 3 4 5
6	1 1 4
7	0 4 5
8	7 8
9	5 9

Find:

a. (1 point) Range

b. (1 point) Mode

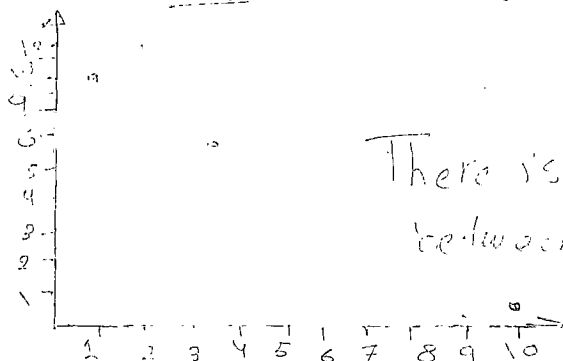**c.** (3 points) The quartiles Q1, Q2 and Q3

d. (3 points) Draw the box plot and comment on the distribution of the data.

3

(41

3. The observations are given for the variables X and Y.

X	Y
3	9
10	1
2	12
1	10

a. (2 points) Draw a scatter plot and comment on your plot.

There is relation between X only.

b. (3 points) Compute the covariance between X and Y and comment on your result.

$$\bar{x} = \frac{3+10+2+1}{4} = 4 \qquad \bar{y} = \frac{9+1+12+10}{4} = 8$$

$$cov(x,y) = \frac{(3-4)(9-8)+(10-4)(1-8)+(2-4)(12-8)+(1-4)(10-8)}{4-1}$$

$$= \frac{-1+(-42)+(-8)+(-6)}{3} = \frac{-57}{3} = -19$$

c. (4 points) Compute the correlation between X and Y and comment on your result.

$$S^2x = \frac{(3-4)^2+(10-4)^2+(2-4)^2+(1-4)^2}{3} = \frac{(50)}{3}$$

$$S x = 7.52\ldots$$

$$S^2 y = \frac{(9-8)^2+(1-8)^2+(12-8)^2+(10-8)^2}{3} = \frac{\ldots}{3}$$

$$Sy = 4.830\ldots$$

$$Corr = \frac{-19}{(4.32\ldots)(4.5304)} = -0.52\ldots$$

There is a strong negative between X and Y

(42

4. (5 points) If $P(A \mid B) = 0.65$, $P(B') = 0.30$ and $P(A) = 0.60$, find $P(A \cup B)$.

5. (4 points) If $P(A') = 0.40$, $P(B) = 0.50$ and $P(A \cap B) = 0.03$. Are A and B independent?

6. (3 points) Find the geometric mean of the numbers 3, 8, 9 and 10. Note that the formula for the geometric mean is $\overline{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n}$

7. (4 points) Suppose that you first toss a coin and then a die. How many different outcomes are possible?

(43

8. A survey is conducted among 400 students about their preference for the class time. 60 % of the participants are females.

Class Time

Gender	Morning	Afternoon	Evening	Total
Male	30			
Female	80	100		
Total		140		

a. (2 points) Fill the table above based on the given information.

b. (2 points) What is the probability that a randomly selected student is male and prefers morning class?

c. (3 points) What is the probability that a randomly selected student is female given that the student prefers afternoon class?

d. (3 points) If a randomly selected student is male, what is the probability that the student prefers evening class?

9. According to the historical data, on average 2.5 workers lose their job every month.

a. (3 points) Find the probability that only 3 workers lose their jobs during the next two months.

Poission :—   New $\lambda = 2.5 \times 2 = 5$ $\lambda = 2.5$

$P(x=3)$

$P(3) = \dfrac{\lambda^x \, e^{-\lambda}}{x!}$

$P(3) = \dfrac{5^3 \, e^{-5}}{3!} = 0.1403$

b. (4 points) Find the probability that at least 2 workers lose their job during a randomly selected month.

$P(x \geq 2) = P(x=2) + P(x=3) + \cdots\cdots P(x=5)$

$= 1 - \Big\{ P(x=0) + P(x=1) \Big\}$

$= 1 - \left\{ \dfrac{2.5^0 \, e^{-2.5}}{0!} + \dfrac{2.5^1 \, e^{-2.5}}{1!} \right\}$

$1 - \Big\{ 0.0820 + 0.1641 \Big\}$

$= 0.7539$

45

10. According to a survey, it is estimated that 45% of the customers prefer to pay cash for their shopping and the rest pay by credit card. Suppose that 8 random customers are sampled.

$n = 8$

a. (3 points) Find the probability that only 5 of the customers pay cash for shopping?

$P(x_{cash} = 5)$

Banical.

Cash 45

Cred 55

$P(x = 5)$
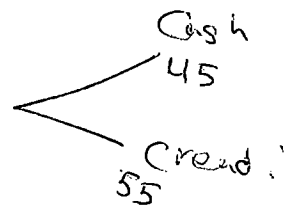
$_8C_5 (0.45)^5 (0.55)^3$

$= 0.17192$

b. (4 points) Find the probability that at most 6 customers pay cash for shopping?

$P(x \leq 6) = \{(x=0) + \ldots (x=6)\}$

$1 - \{(x=7) + (x=8)\}$

$1 - \{_8C_7 (0.45)^7 (0.55)^1 + _8C_8 (0.45)^8 (0.55)^0\}$

$1 - \{0.01644 + 1.68 \times 10^{-3}\}$

$1 - 0.01812 = 0.98188$

c. (3 points) Find the probability that 2 customers pay by credit card.

8

11. Z is a standard normal random variable.

a. (3 points) Find P(-1.32 $\leq$ Z)

b. (3 points) If (Z > k) = 0.7325, find k.

c. (4 points) Given P(- c $\leq$ Z $\leq$ c) = 0.5408, find c.

9

(47

12. Assume that the average annual rainfall (R) in a city is normally distributed with a mean of 12 cm. The variance of annual rainfall is 9.

a. (4 points) Find the probability that the rainfall during a randomly selected year will be between 10cm and 15cm?

b. (4 points) Find t such that $P(R > t) = 0.09$

c. (5 points) If you randomly select 10 years, what is the probability that **average rainfall** will be at least 14 cm?

13. Time to fill a tax form is uniformly distributed between 40 and 90 minutes.

a. (2 points) Write the probability density function and graph it.

b. (2 points) What is the probability that the time to fill a randomly selected tax form will be at most 60 minutes?

c. (2 points) What is the expected time to fill 8 tax files?

d. (2 points) If an accountant charges 500 fills per minute to fill a tax file, what is the expected cost of filling a randomly selected tax file?

(49

Gulf University for Science and Technology

Basic Probability and Statistics

Final Examination – Summer II 2012

Course Code: MATH 121 **Section:.......**

Instructor: Dr. Harun Aydilek   Date: September 6, 2012

Student Name: _____

Student Number: _____KEY._____

Aids Allowed You can use a calculator, but it cannot be shared.

Instructions:

1. The exam is double sided.
2. You can use the formula sheets and tables given to you.

This examination has a cover page, 10 pages for questions, and one blank page at the end for extra space. Total pages is 12 counting double side. In addition, formula sheets and tables are 5 pages in total. Before you start the examination please verify them.

No Questions are allowed during the examination

Student signature: _____

1. Based on the scores of 40 students, the following table is constructed.

a. (4 points) Fill the following table.

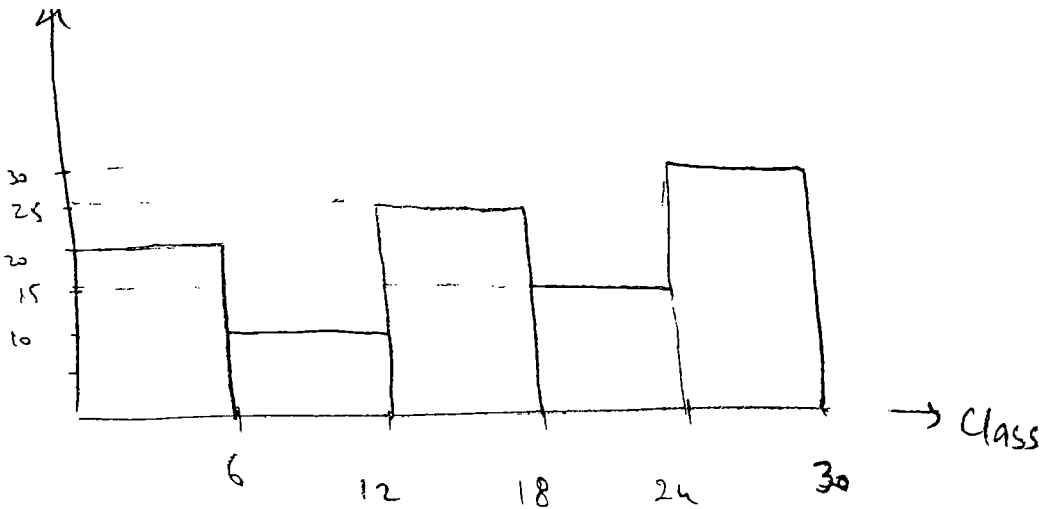| Class	Frequency	Cumulative Frequency	Relative Frequency	Percentage
0 but less than 6	8	8	0.20	20
6 but less than 12	4	12	0.10	10
12 but less than 18	10	22	0.25	25
18 but less than 24	6	28	0.15	15
24 but less than 30	12	40	0.30	30
Total	40	——	1	100

b. (2 points) What percent of the students scored less than 18?

$$20 + 10 + 25 = 55\%$$

OR

$$\frac{8 + 4 + 10}{40} \times 100 = \frac{22}{40} \times 100 = 55\%$$

c. (3 points) Plot the histogram for the percentage.

2

2. Exam 1 scores of 14 students are given below as a stem-and-leaf display.

Stem	Leaves
5	2 3 4 5
6	1 1 4
7	0 4 5
8	7 8
9	5 9

Find:

a. (1 point) Range $\quad max - min = 99 - 52 = 47$

b. (1 point) Mode $\quad 61$

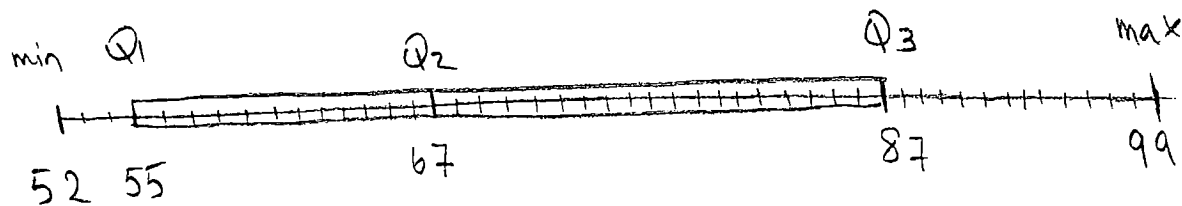**c.** (3 points) The quartiles Q1, Q2 and Q3

Q_1 position $= \dfrac{14+1}{4} = 3.75 \Rightarrow Q_1 = 4^{th}$ data $\Rightarrow \boxed{Q_1 = 55}$

Q_2 position $= 2\left(\dfrac{14+1}{4}\right) = 7.5 \Rightarrow Q_2 = \dfrac{7^{th}\, data + 8^{th}\, data}{2} = \dfrac{64+70}{2} \Rightarrow \boxed{Q_2 = 67}$

Q_3 position $= 3\left(\dfrac{14+1}{3}\right) = 11.25 \Rightarrow Q_3 = 11^{th}$ data $\Rightarrow Q_3 = 87$

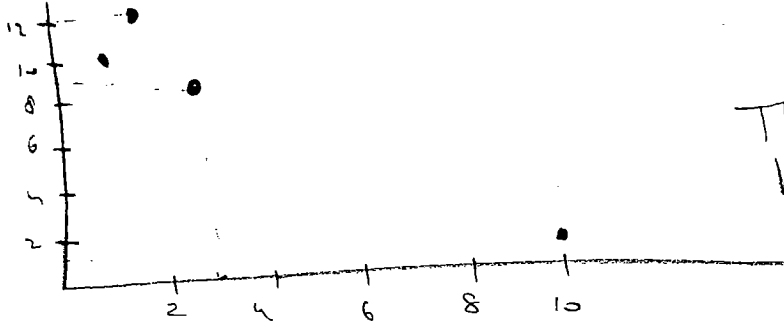**d.** (3 points) Draw the box plot and comment on the distribution of the data.

min Q1 Q2 Q3 max
52 55 67 87 99

Right - skewed.

3

$\left(52 \right.$

3. The observations are given for the variables X and Y.

X	Y
3	9
10	1
2	12
1	10

a. (2 points) Draw a scatter plot and comment on your plot.

There is (−) relation between X & Y.

b. (3 points) Compute the covariance between X and Y and comment on your result.

$$\overline{X} = \frac{3+10+2+1}{4} = \frac{16}{4} = 4 \qquad \overline{Y} = \frac{9+1+12+10}{4} = \frac{32}{4} = 8$$

$$cov(X,Y) = \frac{(3-4)(9-8)+(10-4)(1-8)+(2-4)(12-8)+(1-4)(10-8)}{4-1}$$

$$= \frac{(-1)+(-42)+(-8)+(-6)}{3} = \frac{-57}{3} = -19 \Rightarrow \text{There is } (-) \text{ relation between X & Y.}$$

c. (4 points) Compute the correlation between X and Y and comment on your result.

$$S^2_X = \frac{(3-4)^2+(10-4)^2+(2-4)^2+(1-4)^2}{4-1} = \frac{1+36+4+9}{3} = \frac{50}{3} = 16.67$$

$$S_X = \sqrt{16.67} = 4.08$$

$$S^2_Y = \frac{(9-8)^2+(1-8)^2+(12-8)^2+(10-8)^2}{4-1} = \frac{1+49+16+4}{3} = \frac{70}{3} = 23.33$$

$$S_Y = \sqrt{23.33} = 4.83$$

$$Corr(X,Y) = \frac{cov(X,Y)}{S_X \cdot S_Y} = \frac{-19}{(4.08)(4.83)} = -0.96 \Rightarrow \text{There is a very strong & negative relation between X & Y}$$

53

4. (5 points) If P(A | B) = 0.65, P(B') = 0.30 and P(A) = 0.60, find P(A U B).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.60 + 0.70 - P(A \cap B)$$

$$= 0.60 + 0.70 - 0.455$$

$$P(A \cup B) = 0.845$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$0.65 = \frac{P(A \cap B)}{0.70}$$

$$0.65 \times 0.70 = P(A \cap B)$$

$$0.455 = P(A \cap B)$$

5. (4 points) If P(A') = 0.40, P(B) = 0.50 and P(A ∩ B) = 0.03. Are A and B independent?

$$P(A) = 0.60$$
$$P(B) = 0.50$$

$$P(A \cap B) = 0.03 \qquad P(A) \cdot P(B) = 0.60 \times 0.50 = 0.30$$

$$P(A \cap B) \neq P(A) \cdot P(B) \Rightarrow \text{No; they are not independent.}$$
$$0.03 \neq 0.30$$

6. (3 points) Find the geometric mean of the numbers 3, 8, 9 and 10. Note that the formula for the geometric mean is $\overline{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n}$

$$\overline{X}_G = \left(3 \times 8 \times 9 \times 10\right)^{1/4} = 6.82$$

7. (4 points) Suppose that you first toss a coin and then a die. How many different outcomes are possible?

Coin   die
$$\underline{\quad 2 \quad} \times \underline{\quad 6 \quad} = \underline{\underline{12}}$$

64

8. A survey is conducted among 400 students about their preference for the class time. 60 % of the participants are females.

0.60 × 400
= 240
— females

Class Time

Gender	Morning	Afternoon	Evening	Total
Male	30	40	90	160
Female	80	100	60	240
Total	110	140	150	400

a. (2 points) Fill the table above based on the given information.

b. (2 points) What is the probability that a randomly selected student is male and prefers morning class?

$$P(\text{male} \cap \text{Morning}) = \frac{30}{400} = 0.075$$

c. (3 points) What is the probability that a randomly selected student is female given that the student prefers afternoon class?

$$P(\text{Female} \mid \text{Afternoon}) = \frac{P(\text{Female} \cap \text{Afternoon})}{P(\text{Afternoon})} = \frac{100/400}{140/400} = 0.7143$$

OR (directly) $\frac{100}{140} = 0.7143$

d. (3 points) If a randomly selected student is male, what is the probability that the student prefers evening class?

$$P(\text{Evening} \mid \text{Male}) = \frac{P(\text{Evening} \cap \text{Male})}{P(\text{Male})} = \frac{90/400}{160/400} = 0.5625$$

OR (directly) $= \frac{90}{160} = 0.5625$

6

55

9. According to the historical data, on average 2.5 workers lose their job every month.

$\wedge = 2.5$ per month

a. (3 points) Find the probability that only 3 workers lose their jobs during the next two months.

new $\lambda = 2.5 \times 2 = 5$

$$P(x=3) = \frac{e^{-5} \cdot 5^3}{3!}$$

$$= 0.1404$$

b. (4 points) Find the probability that at least 2 workers lose their job during a randomly selected month.

$\lambda = 2.5$

$$P(X \geq 2) = P(X=2) + P(X=3) + \cdots + P(X=\infty)$$

$$= 1 - \{ P(X=0) + P(X=1) \}$$

$$= 1 - \left\{ e^{-2.5} \frac{(2.5)^0}{0!} + e^{-2.5} \frac{(2.5)^1}{1!} \right\}$$

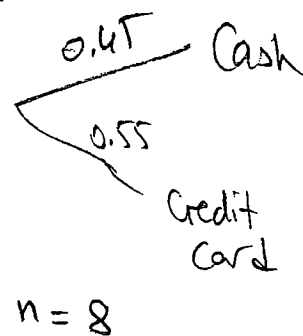$$= 1 - \{ 0.0821 + 0.2052 \}$$

$$= 1 - \{ 0.2873 \}$$

$$= 0.7127$$

7

56

10. According to a survey, it is estimated that 45% of the customers prefer to pay cash for their shopping and the rest pay by credit card. Suppose that 8 random customers are sampled. (Binomial)

a. (3 points) Find the probability that only 5 of the customers pay cash for shopping?

$$P(X=5) = {}_8C_5 \, (0.45)^5 (0.55)^3$$

$$= 0.1719$$

$$0.45 - Cash$$
$$0.55 - Credit\ Card$$

$$n = 8$$

b. (4 points) Find the probability that at most 6 customers pay cash for shopping?

$$P(X \le 6) = P(X=0) + \ldots + P(X=6)$$

$$= 1 - \{ P(X=7) + P(X=8) \}$$

$$= 1 - \{ {}_8C_7 (0.45)^7 (0.55)^1 + {}_8C_8 (0.45)^8 (0.55)^0 \}$$

$$= 1 - \{ 0.0164 + 0.0017 \}$$

$$= 1 - 0.0181$$

$$= 0.9819$$

c. (3 points) Find the probability that 2 customers pay by credit card.

$$P(Y=2) = {}_8C_2 (0.55)^2 (0.45)^6 = 0.0703$$

_OR_ 2 credit card = 6 cash

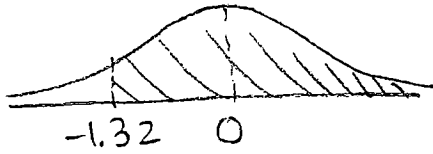$$\Rightarrow P(X=6) = {}_8C_6 (0.45)^6 (0.55)^2 = 0.0703$$

8

(57

11. Z is a standard normal random variable.

a. (3 points) Find P(-1.32 ≤ Z)

$$= 1 - 0.0934$$
$$= 0.9066$$

−1.32   0

b. (3 points) If (Z > k) = 0.7325, find k.

k > 0   k < 0   0.7325

⇒

$$= 1 - 0.7325$$
$$= 0.2675$$
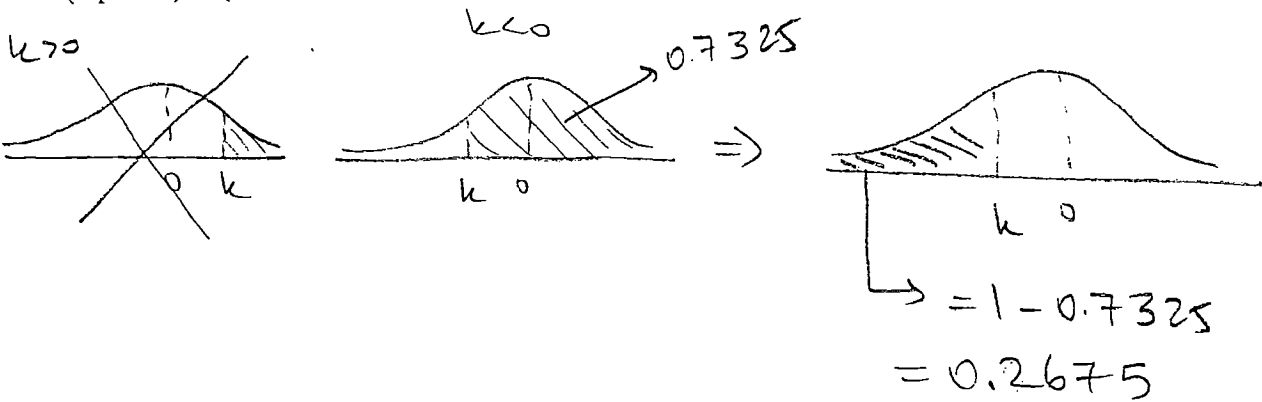
↪ seach (−) table for 0.2675 or closest ⇒ k = −0.62

c. (4 points) Given P(- c ≤ Z ≤ c) = 0.5408, find c.
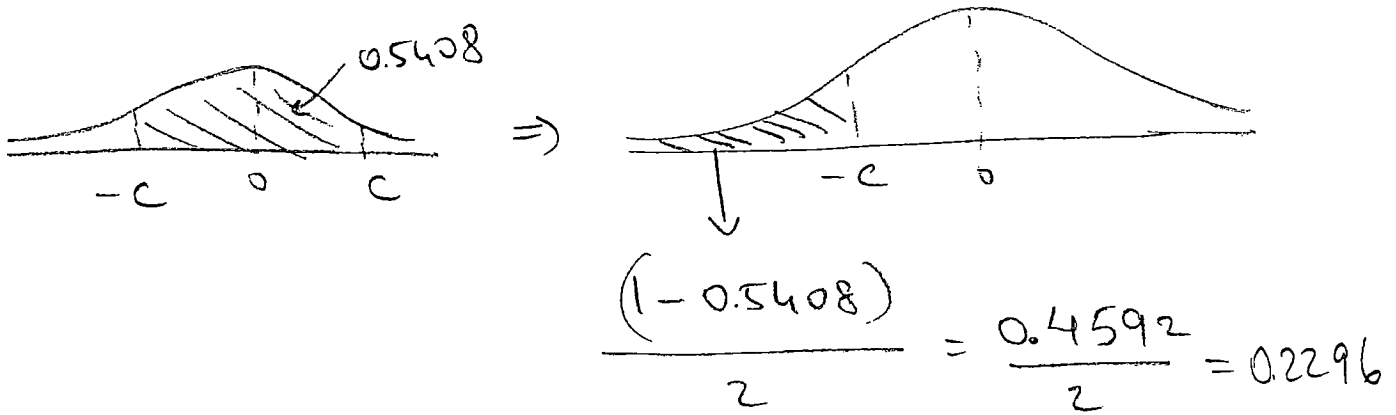
0.5408   −c   0   c   ⇒   −c   0

$$\frac{(1 - 0.5408)}{2} = \frac{0.4592}{2} = 0.2296$$

$$\Rightarrow -c = -0.74 \Rightarrow c = 0.74$$

9

58

12. Assume that the average annual rainfall (R) in a city is normally distributed with a mean of 12 cm. The variance of annual rainfall is 9. $\Rightarrow \mu = 12 \,;\, \sigma^2 = 9 \,;\, \sigma = 3$

a. (4 points) Find the probability that the rainfall during a randomly selected year will be between 10cm and 15cm?
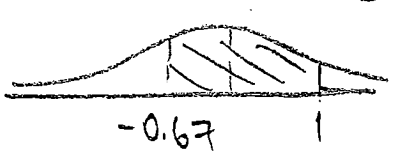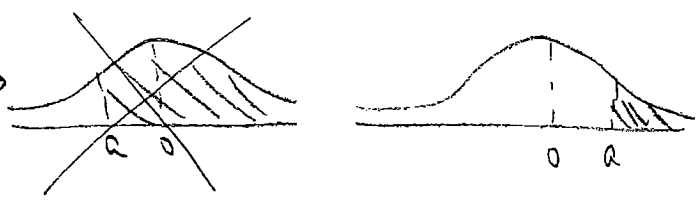
$$P(10 < X < 15) = P\left(\frac{10-12}{3} < z < \frac{15-12}{3} \right) = P\left(-\frac{2}{3} < z < \frac{3}{3} \right)$$

$$z = \frac{X - \mu}{\sigma} \quad\Big| \quad = P(-0.67 < z < 1) =$$

$$= 0.8413 - 0.2514$$

$$= 0.5899$$

-0.67   1

b. (4 points) Find t such that P(R > t) = 0.09

$$P\left(z > \left(\frac{t-12}{3}\right) \right) = 0.09 \longrightarrow$$

$$P(z > a) = 0.09$$

0   a

$$\Rightarrow \qquad = 1 - 0.09 = 0.91$$

$$\Rightarrow \quad a = 1.34$$

$$a = \frac{t-12}{3} \Rightarrow 1.34 = \frac{t-12}{3} \Rightarrow t = 12 + 3(1.34) = \underline{16.02}$$

c. (5 points) If you randomly select 10 years, what is the probability that **average rainfall** will be at least 14 cm? (SAMPLING) $n = 10$

Standard error $= \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

$$\sigma_{\bar{X}} = \frac{3}{\sqrt{10}} = 0.95$$

$$P(\bar{X} > 14)$$

$$P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} > \frac{14 - \mu}{\sigma_{\bar{X}}} \right) = P\left(z > \frac{14 - 12}{0.95} \right)$$

$$= P(z > 2.11) = 1 - 0.9826 = 0.0174$$

10

0   2.11

$\left(59\right.$

13. Time to fill a tax form is uniformly distributed between 40 and 90 minutes.

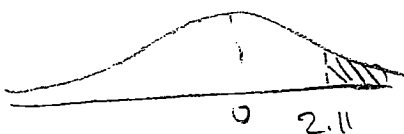**a.** (2 points) Write the probability density function and graph it.

$$f(x) = \begin{cases} \frac{1}{50} & \text{if } 40 \leq x \leq 90 \\ 0 & \text{otherwise} \end{cases}$$
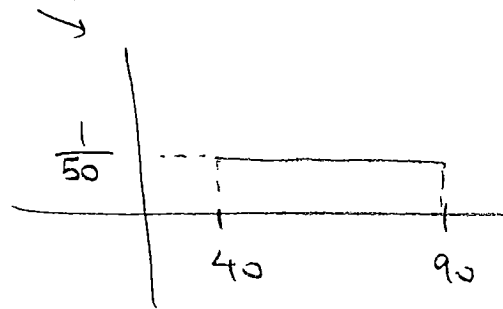
b. (2 points) What is the probability that the time to fill a randomly selected tax form will be at most 60 minutes?

$$P(x < 60)$$
$$= 20 \times \frac{1}{50} = 0.40$$
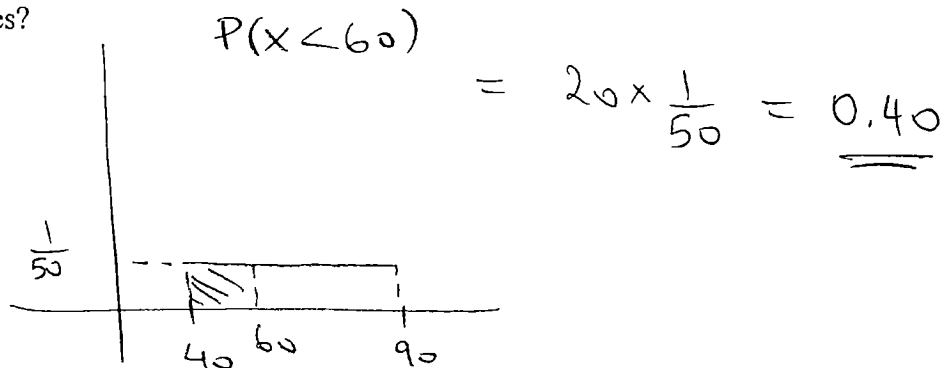
c. (2 points) What is the expected time to fill 8 tax files?

$$E(x) = \frac{40+90}{2} = 65 \text{ minutes}$$

for 8 $\longrightarrow$ $8 \times 65 = 520$ minutes.

d. (2 points) If an accountant charges 500 fills per minute to fill a tax file, what is the expected cost of filling a randomly selected tax file?

$$E(x) = 65 \text{ min}$$

Expected cost $= 65 \times 500 = 32500$ fills
$$= 32.5 \text{ KD}$$

11

(60

Fall 2012

- Quizzes and Solutions

- Exam1 and Solutions

- Midterm exam and solutions

- Exam2 and Solutions

- Final exam and solutions

(61

1) Review test

2) Evaluate

a) $1 - (0.1971 + 0.4219) =$

b) $65\% \times 72\% \quad =$

c) $(0.35)^2 \times (0.65)^7 \quad =$

d) $0.4 \times 4 + 0.1 \times 2 + 0.5 \times 8 =$

e) $2 \times (8-4)^2 + 3 \times (12-4)^2 \quad =$

f) $\sqrt{8.14} \quad =$

g) $\sqrt[3]{4 \times 7 \times 6} \quad =$

h) $0.80 + 0.1 =$

i) $\dfrac{12!}{5!} + 0! =$

j) $\dfrac{e^{-3} \cdot 3^4}{4!} =$

k) Find 25% of 480

(62

l) $\dfrac{85}{160} = $ (use. 4 decimals)

m) $_8C_3 = $

n) 25 % of a number is 60. Find the number.

o) Write the following expressions by using math notations.

example: i) more than 5 $\Rightarrow$ $x > 5$

ii) at most 7 $\Rightarrow$

iii) at least 4 $\Rightarrow$

iv) less than 8 $\Rightarrow$

v) 3 or more $\Rightarrow$

vi) no more than 9 $\Rightarrow$

vii) greater than 10 $\Rightarrow$

(63

Name :

ID#

1) Review test

2) Evaluate

a) $1 - (0.1971 + 0.4219) = 1 - 0.6190 = 0.3810$

b) $65\% \times 72\% = 0.65 \times 0.72 = 0.4680$

c) $(0.35)^{3} \times (0.65)^{7} = 0.0060$

d) $0.4 \times 4 + 0.1 \times 2 + 0.5 \times 8 = 5.8$

e) $2 \times (8-4)^{2} + 3 \times (12-4)^{2} = 224$

f) $\sqrt{8.14} = 2.8531$

g) $\sqrt[3]{4 \times 7 \times 6} = 5.5178$

h) $0.80 + 0.1 = 0.90$

i) $\dfrac{12!}{5!} + 0! = 3991680 + 1 = 3991681$

j) $\dfrac{e^{-3} \cdot 3^{4}}{4!} = 0.1680$

k) Find 25% of $480 = 0.25 \times 480 = 120$

(64

l) $\dfrac{85}{160} = 0.5313$ (use 4 decimals)

m) $_8C_3 = 56$

n) 25 % of a number is 60. Find the number. $= \dfrac{60}{0.25} = 240$

o) Write the following expressions by using math notations.

example; i) more than 5 $\Rightarrow$ $x > 5$

ii) at most 7 $\Rightarrow$ $x \leq 7$

iii) at least 4 $\Rightarrow$ $x \geq 4$

iv) less than 8 $\Rightarrow$ $x < 8$

v) 3 or more $\Rightarrow$ $x \geq 3$

vi) no more than 9 $\Rightarrow$ $x \leq 9$

vii) greater than 10 $\Rightarrow$ $x > 10$

(65

Math 121 Fall 2012 Quiz #1

Name:..........................
ID# : ....

1. Consider the data set 37, 52, 33, 48, 44, 90, 42, 60 for the following parts.

   a. Find the quartiles Q1, Q2 and Q3.

   b. List 5-number summary.

   c. Draw Box-Plot.

   d. Comment on the distribution of the data based on the Box-Plot.

1

(66

2. For the given data set,

X	Y
5	3
9	1
4	8

a. **Find** the covariance between X and Y, and **comment** on your result.

b. **Find** the sample correlation coefficient between X and Y, and **comment** on your result.

2

(67

Math 121 Fall 2012 Quiz #1

Name:....Key........

ID# : ....

1. Consider the data set 37, 52, 33, 48, 44, 90, 42, 60 for the following parts.

   a. Find the quartiles Q1, Q2 and Q3. $n = 8$

   $$33, 37, 42, 44, 48, 52, 60, 90$$

   Q_1 position $= \frac{n+1}{4} = \frac{8+1}{4} = 2.25 \Rightarrow Q_1 = 2^{nd}$ data $\Rightarrow Q_1 = 37$

   Q_2 position (median) $= 2\left(\frac{n+1}{4}\right)$ $\left(OR \ \frac{n+1}{2}\right) = 4.5 \Rightarrow Q_2 = \frac{4^{th} data + 5^{th} data}{2}$

   $\Rightarrow Q_2 = \frac{44 + 48}{2} = 46.$

   Q_3 position $= 3\left(\frac{n+1}{4}\right) = 3\left(\frac{9}{4}\right) = 6.75 \Rightarrow Q_3$ is 7th data

   $\Rightarrow Q_3 = 60$

   b. List 5-number summary.

   $min = 33$
   $Q_1 = 37$
   $Q_2 = 46$
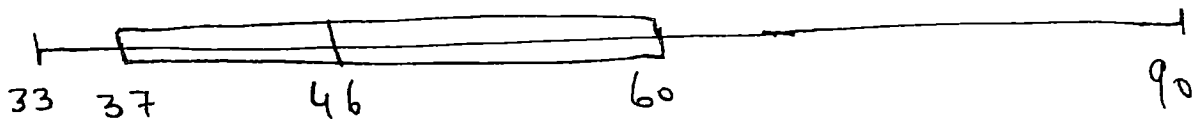 $Q_3 = 60$
   $max = 90$

   c. Draw Box-Plot.

   33   37   46   60   90

   d. Comment on the distribution of the data based on the Box-Plot.

   The data is Right Skewed.

1

(68

2. For the given data set,

$$n = 3.$$

X	Y
5	3
9	1
4	8

$$\overline{X} = \frac{5+9+4}{3} = \frac{18}{3} = 6$$

$$\overline{Y} = \frac{3+1+8}{3} = \frac{12}{3} = 4$$

a. Find the covariance between X and Y, and **comment** on your result.

$$Cov(X,Y) = \frac{(5-6)\cdot(3-4) + (9-6)\cdot(1-4) + (4-6)\cdot(8-4)}{3-1}$$

$$= \frac{1 + (-9) + (-8)}{2} = -\frac{16}{2} = -8 < 0$$

$$Cov(X,Y) < 0 \Rightarrow \text{There is a negative linear relation between } X \& Y.$$

b. Find the sample correlation coefficient between X and Y, and **comment** on your result.

$$Correlation = \frac{Cov(X,Y)}{S_X \cdot S_Y}$$

$$S_X^2 = \frac{(5-6)^2 + (9-6)^2 + (4-6)^2}{3-1} = \frac{1+9+4}{2} = \frac{14}{2} = 7$$

$$S_X = \sqrt{7} = 2.646$$

$$S_Y^2 = \frac{(3-4)^2 + (1-4)^2 + (8-4)^2}{3-1} = \frac{1+9+16}{2} = \frac{26}{2} = 13$$

$$S_Y = \sqrt{13} = 3.606$$

$$Corr = \frac{Cov(X,Y)}{S_X \cdot S_Y} = \frac{-8}{(2.646)(3.606)} = -0.84$$

69

$\Rightarrow$ There is a strong negative linear relation between $X \& Y$.

Math 121   Fall 2012 Quiz 2   Name: **ID # :**

X	4	8	16
P(X)	0.1	0.6	0.3

1) (30 points) Probability density function for X is given.

a) Find the missing probability and then find the   expected value, E(X).

$$= 1 - \{0.1 + 0.3\} = 0.6$$

$$E(x) = 4(0.1) + 8(0.6) + 16(0.3) = 0.4 + 4.8 + 4.8 = 10$$

b) Find the standard deviation of X.

$$Var(x) = (4-10)^2 0.1 + (8-10)^2 0.6 + (16-10)^2 0.3$$

$$= 3.6 + 2.4 + 10.8 = 16.8$$

$$St.dav = \sqrt{16.8} = 4.09878$$

2) (40 points) Food Delivery Company receives on average 2.5 orders per hour. What is the probability that the company receives **Poission** $\lambda = 2.5$

a) three orders in one hour?

$$\rightarrow new \to x = 3 \to 2.5 =$$

$$P(x = 3)$$

$$= \frac{\lambda^x e^x}{x!} = \frac{2.5^3 e^{-2.5}}{3!} = 0.21376$$

b) four orders in two hours?

$$P(x = 4) \qquad \rightarrow New \; \lambda = 2.5 \times 2 = 5$$

$$= \frac{5^4 e^{-5}}{4!} = 0.17546$$

3) (30 points) According to a survey, the probability of a student passing the statistics course is 0.7. If 12 students are randomly selected, what is the probability that at most 10 students pass the course?

Binomial $n = 12$ passing
                                        0.7
                                        0.3 not passing

$$P(x \leq 10) = P(x = 0) + \cdots + P(x = 10)$$

$$= 1 - \{P(x = 11) + P(x = 12)\}$$

$$= 1 - \{{}_{12}C_{11}(0.7)^{11}(0.3)^{11-12} + {}_{12}C_{12}(0.7)^{12}(0.3)^{12-12}\} \quad (7-0$$

$$= 1 - \{0.0712 + 0.0138\} = 1 - 0.0850 = 0.915$$

Math 121   Fall 2012 Quiz 2   Name: ID # :

1) (30 points) Probability density function for X is given.

X	4	8	16
P(X)	0.1	o.6	0.3

a) Find the missing probability and then find the expected value, E(X). $= 1 - \{0.1 + 0.3\} = 1 - 0.4 = 0.6$

$E(x) = 4 \times 0.1 + 8 \times 0.6 + 16 \times 0.3 = 0.4 + 4.8 + 4.8 = 10.$

b) Find the standard deviation of X.

$Var(X) = (4-10)^2 \times 0.1 + (8-10)^2 \times 0.6 + (16-10)^2 \times 0.3$

$= 3.6 + 2.4 + 10.8 = 16.8$

$St. dev = \sqrt{16.8} = 4.099$

2) (40 points) Food Delivery Company receives on average 2.5 orders per hour. What is the probability that the company receives poisson ; $\lambda = 2.5$ per hour.

a) three orders in one hour? $\lambda = 2.5$

$P(X=3) = \dfrac{e^{-2.5} \cdot (2.5)^3}{3!} = 0.2138$

b) four orders in two hours? new $\lambda = 2.5 \times 2 = 5$

$P(Y=4) = e^{-5} \cdot \dfrac{5^4}{4!} = 0.1755$

3) (30 points) According to a survey, the probability of a student passing the statistics course is 0.7. If 12 students are randomly selected, what is the probability that at most 10 students pass the course?

Binomial ; n = 12, pass 0.7 fail 0.3

$P(at\ most\ 10) = P(X \le 10) = P(X=0) + \cdots + P(X=10)$

$= 1 - \{P(X=11) + P(X=12)\} = 1 - \{ _{12}C_{11}(0.7)^{11}(0.3)^1 + _{12}C_{12}(0.7)^{12}(0.3)^0 \}$

$= 1 - \{0.0712 + 0.0138\} = 1 - 0.0850 = 0.915$ (73)

1) Find

a) $P(-1.12 < z < 1.43)$

b) $P(z > 1.08)$

2) a) $P(z < a) = 0.1711 \Rightarrow a = ?$

b) $P(z > b) = 0.9808 \Rightarrow b = ?$

c) $P(z > c) = 0.1228 \Rightarrow c = ?$

(72

St. deviation = 2

a) $P(X > 18)$

b) $P(X < k) = 0.80 \Rightarrow k = ?$
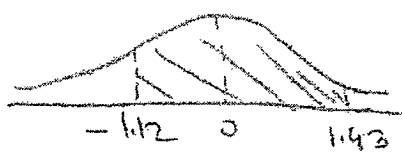
i.e 80% of the dogs will weight less than how many kg?

90% of the dogs will weight more than how many kg?

)se the numbers in part Question 3). $X \sim$ normal $\mu = 15$
$\sigma = 2$.

you randomly select 16 dogs, what is the
ability that the sample mean will be more than
5 kg?

(73

1) Find

a) $P(-1.12 < Z < 1.43) =$

$= 0.9236 - 0.1314 = 0.7922$

b) $P(Z > 1.08)$
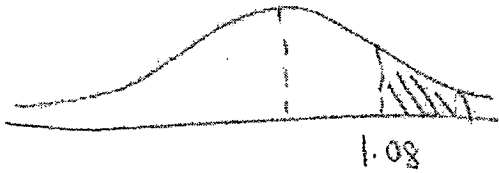
$= 1 - 0.8599 = 0.1401$

2) a) $P(Z < a) = 0.1711 \Rightarrow a = ?$
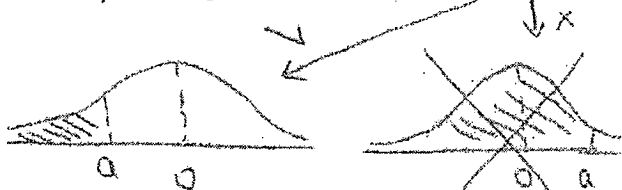
$a = -0.95$

b) $P(Z > b) = 0.9808 \Rightarrow b = ?$

$\Rightarrow 1 - 0.9808 = 0.0192$

$\Rightarrow (-) \text{ table}$

$\Rightarrow b = -2.07$

c) $P(Z > c) = 0.1228 \Rightarrow c = ?$
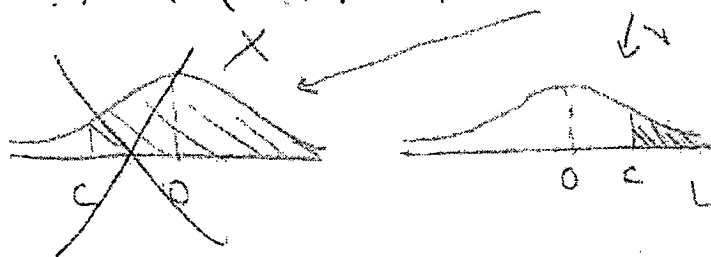
$\Rightarrow 1 - 0.1228 = 0.8772$

$\Rightarrow (+) \text{ table}$

$\Rightarrow c = 1.16$

(closest)

74

... , ... , ST. DEVIATION = 2
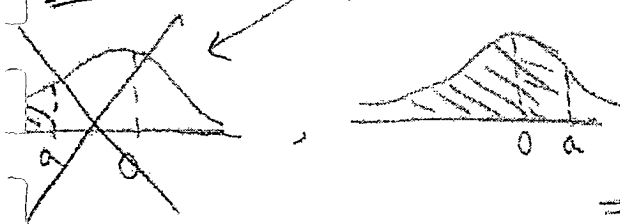
1) $P(X > 18) = P\left(\dfrac{\overset{Z}{\overbrace{X-\mu}}}{\sigma} > \dfrac{18-\mu}{\sigma}\right) = P\left(Z > \dfrac{18-15}{2}\right)$

$= P(Z > 1.5) =$ $= 1 - 0.9332$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 0.0668$
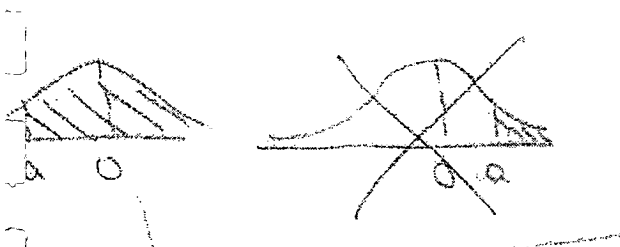
2) $P(X < k) = 0.80 \Rightarrow k = ?$ look for a s.t $P(Z < a) = 0.80$

i.e 80% of the dogs will weight less than how many kg?

$\Rightarrow a = 0.84$ (closest)

$\Rightarrow k = \sigma \cdot a + \mu$

$\qquad = (2)(0.84) + 15$

$\qquad = 1.68 + 15 = 16.68$

90% of the dogs will weight more than how many kg?

$(X > k) = 0.90 \longrightarrow$ look for a s.t. $P(Z > a) = 0.90$

$\Rightarrow 1 - 0.90 = 0.10$

$\Rightarrow (-)$ table $\Rightarrow a = -1.28$ (closest)

$\Rightarrow k = \sigma \cdot a + \mu = (2)(-1.28) + 15$

$\qquad = -2.56 + 15 = 12.44$

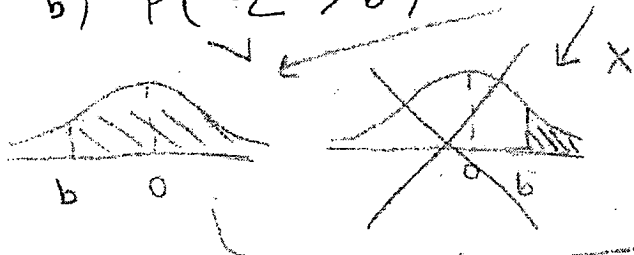3) se the numbers in part Question 3). $X \sim$ normal $\mu = 15$ $\sigma = 2$.

$\qquad\qquad\qquad\qquad n = 16$

you randomly select 16 dogs, what is the
ability that the sample mean will be more than

5 kg? $P(\overline{X} > 16.5) = P\left(\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} > \dfrac{16.5 - 15}{2/\sqrt{16}}\right) = P(Z > 3)$

standard $\longleftarrow$ $\qquad\qquad\qquad\qquad = 1 - 0.9987 = 0.0013$

(75

Math 121 Basic Probability and Statistics
Fall 2012 Exam 1

Name :_____   ID # :_____

Show all your work to get full credit.

1. The temperatures for 20 random days are given below as a stem-and-leaf display.

Stem	Leaves
2	2 3 4 5 6
3	0 1 1 2 3 5 8 9
4	0 0 1 1 2 2 3

    a. (15 points) Fill the following table.

Class	Frequency	Cumulative Frequency	Relative Frequency	Percentage
20 but less than 26				
26 but less than 32				
32 but less than 38				
38 but less than 44				
Total				

    b. (5 points) What percent of the data are greater than 38?

    c. (5 points) Plot the histogram for the frequency.

1

76

2. Consider the data set 18, 24, 26, 27, 27, 20, 11, 21, 30, 31 for the following questions.

    a. (5 point) Find the median.

    b. (10 points) Find the quartiles Q1 and Q3.

    c. (5 points) Draw the box plot and comment on the distribution of the data.

    d. (5 points) Construct a stem and leaf display.

2

$\left(77 \right.$

3. According to a survey among the students, the following data is obtained where X is the number of hours a student studied for the exam and Y is the overall score.

a. (5 points) **Draw** a scatter plot and **comment** on your plot.

X	Y
5	7
1	2
6	10
4	5

b. (10 points) **Compute** the covariance between X and Y and **comment** on your result.

c. (10 points) **Compute** the correlation between X and Y and **comment** on your result.

3

(78

4. For the table on the right,

Class	Frequency
6 – 14	5
14 – 22	8
22 – 30	11

a. (10 points) find the approximate mean

b. (10 points) find the approximate standard deviation.

c. (5 points) Is 29 an outlier? Explain your reason.

4

79

Math 121 Basic Probability and Statistics
Fall 2012 Exam 1

Name : _____   ID # : _____ Key _____

Show all your work to get full credit.

1. The temperatures for 20 random days are given below as a stem-and-leaf display.

<u>Stem   Leaves</u>

2	2 3 4 5 6
3	0 1 1 2 3 5 8 9
4	0 0 1 1 2 2 3

a. (15 points) Fill the following table.

Class	Frequency	Cumulative Frequency	Relative Frequency	Percentage
20 but less than 26	4	4	0.20	20
26 but less than 32	4	8	0.20	20
32 but less than 38	3	11	0.15	15
38 but less than 44	9	20	0.45	45
Total	20		1	100

b. (5 points) What percent of the data are greater than 38?

$$\frac{8}{20} \times 100 = 40\%$$

c. (5 points) Plot the histogram for the frequency.
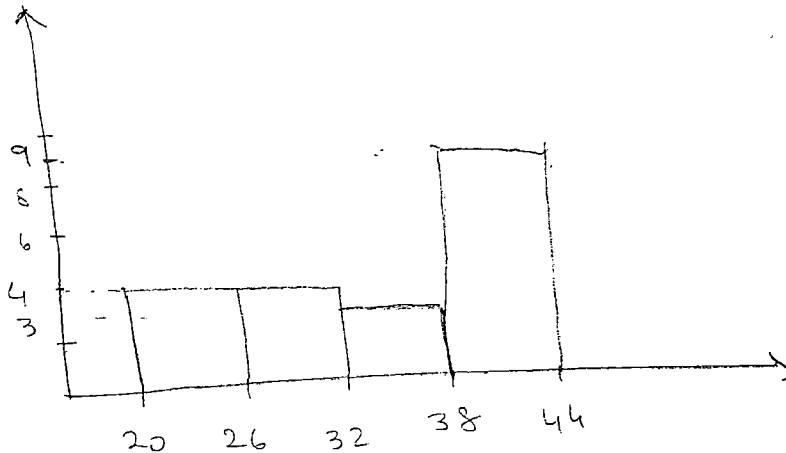
1
(80

2. Consider the data set 18, 24, 26, 27, 27, 20, 11, 21, 30, 31 for the following questions.

$$11, 18, 20, 21, 24, 26, 27, 27, 30, 31 \Rightarrow n = 10.$$

a. (5 point) Find the median.

$$\text{median} = Q_2 \Rightarrow Q_2 \text{ position} = 2\left(\frac{n+1}{4}\right) \left(= OR \ \frac{n+1}{2}\right) = 2\left(\frac{11}{4}\right) = 5.5$$

$$\text{median} = \frac{5^{th} \text{data} + 6^{th} \text{data}}{2} = \frac{24 + 26}{2} = 25.$$

b. (10 points) Find the quartiles Q1 and Q3.

$$Q_1 \text{ position} = \frac{n+1}{4} = \frac{11}{4} = 2.75 \Rightarrow Q_1 = 3^{rd} \text{data}$$
$$Q_1 = 20.$$

$$Q_3 \text{ position} \ 3\left(\frac{n+1}{4}\right) = 3\left(\frac{11}{4}\right) = 8.25 \Rightarrow Q_3 = 8^{th} \text{data}$$
$$Q_3 = 27.$$

$$\& \ min = 11, \ max = 31$$

c. (5 points) Draw the box plot and comment on the distribution of the data.

11   20   25   27   31
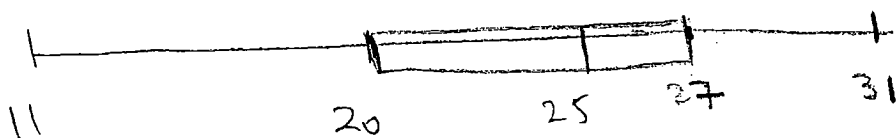
left - skewed .

d. (5 points) Construct a stem and leaf display.

1	1 8
2	0 1 4 6 7 7
3	0 1

2

(81

3. According to a survey among the students, the following data is obtained where X is the number of hours a student studied for the exam and Y is the overall score.

a. (5 points) **Draw** a scatter plot and **comment** on your plot.

$n = 4$

X	Y
5	7
1	2
6	10
4	5
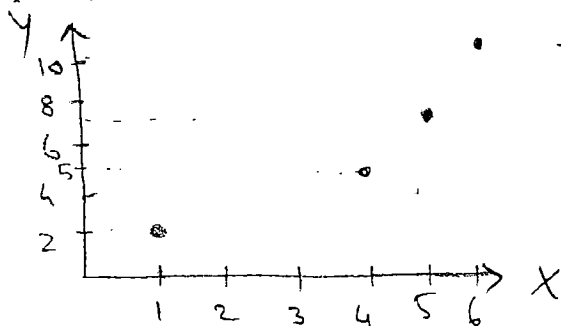
There is positive linear relation between X & Y.

$\overline{X} = \frac{16}{4} = 4$

$\overline{Y} = \frac{24}{4} = 6.$

b. (10 points) **Compute** the covariance between X and Y and **comment** on your result.

$$\text{Cov}(X,Y) = \frac{(5-4)(7-6) + (1-4)(2-6) + (6-4)(10-6) + (4-4)(5-6)}{4-1}$$

$$= \frac{1 + 12 + 8 + 0}{3} = \frac{21}{3} = 7 > 0$$

$\Rightarrow$ There is positive linear relation between X & Y.

c. (10 points) **Compute** the correlation between X and Y and **comment** on your result.

$$S^2_X = \frac{(5-4)^2 + (1-4)^2 + (6-4)^2 + (4-4)^2}{4-1}$$

$$S^2_X = \frac{1 + 9 + 4 + 0}{3} = \frac{14}{3} = 4.67 \Rightarrow S_X = \sqrt{4.67} = 2.16$$

$$S^2_Y = \frac{(7-6)^2 + (2-6)^2 + (10-6)^2 + (5-6)^2}{4-1} = \frac{1 + 16 + 16 + 1}{3}$$

$$S^2_Y = \frac{34}{3} = 11.33 \Rightarrow S_Y = \sqrt{11.33} = 3.37$$

$$\text{Corr}(X,Y) = \frac{7}{(2.16)(3.37)} = 0.96 \Rightarrow \text{There is very strong & negative linear relation btw X & Y.}$$

3 _

(82

2. Given the data sample: 5, 9, 10, 7, 8, 9

  a. **(3 points)** Compute the mean, median and mode. $5, 7, 8, 9, 9, 10$

mean $= x = \frac{5+7+8+9+9+10}{6} = 8$

median $= \frac{8+9}{2} = 8.5$

mode $= 9$

  b. **(2 points)** Compute the standard deviation and the coefficient of variation.

$$S^2 = \frac{(5-8)^2+(7-8)^2+(8-8)^2+(9-8)^2(9-8)^2+(10-8)^2}{6-1} = \frac{16}{5} = 3.2$$

$S = \sqrt{3.2} = 1.7885$

  c. **(2 points)** Find whether 16 an outlier or not.

$Z-scor = \frac{16-8}{1.7885} = 4.4730$

  d. **(4 points)** Compute the range, Q1, Q3 and IQR. $Rang = 10-5 = 5$

Q_1 posion $= \frac{6+1}{4} = 1.75 \Rightarrow Q_1 = 2^{rd} = \boxed{Q_1 = 7}$

Q_3 posion $= 3(\frac{7}{4}) = 5.25 \Rightarrow Q_3 = 5^{th} \boxed{Q_3 = 9}$

$Q_3 - Q_1 = 9 - 7 = 2$

  e. **(2 points)** Draw a box plot displaying the five number summary and comment on the shape of the distribution.

Skewed _ left

$\binom{85}{}$

3. A sample of 300 men is selected. The contingency table below shows their registration status and their preferred source of information on current events. If an man is selected at random,

		Preferred source of information			
		Television	Newspapers	Radio	Internet
Voting registration status	Registered	45	30	45	36
	Not registered	35	44	45	20

a. **(2 points)** What is the probability that he prefers to get his current information from the newspapers?

b. **(2 points)** What is the probability that he is not a registered voter and prefers to get his current information from the internet?

c. **(3 points)** What is the probability that he is a registered voter or prefers to get his current information from the television?

d. **(3 points)** What is the probability that he is a registered voter given that he prefers to get his current information from the radio?

86

4. For the table on the right,

Class	Frequency
6 – 14	5
14 – 22	8
22 – 30	11

midpoint	Freq
10	5
18	8
26	11

24

a. (10 points) find the approximate mean

$$\bar{X} = \frac{5 \times 10 + 8 \times 18 + 11 \times 26}{24} = \frac{50 + 144 + 286}{24} = 20$$

b. (10 points) find the approximate standard deviation.

$$S^2 = \frac{5 \times (10-20)^2 + 8(18-20)^2 + 11(26-20)^2}{24-1} =$$

variance

$$= \frac{500 + 32 + 396}{23} = 40.35$$

Standard dev

$$\rightarrow S = \sqrt{40.35} = 6.35$$

c. (5 points) Is 29 an outlier? Explain your reason.
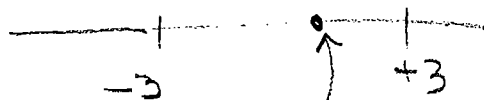
$$Z\text{-Score} = \frac{29-20}{6.35} = 1.42$$

No; it is not an outlier

83

Name: ....

GUST ID #: ....

		50

MIDTERM

1. Given the following table.

Class	Cumulative frequency	Frequency	Relative Frequency	Percent frequency	Cumulative percent frequency
15 but less than 20	4	4	$\frac{4}{15} = 0.27$	27	27
20 but less than 25	7	3	0.20	20	47
25 but less than 30	9	2	0.13	13	60
30 but less than 35	15	6	0.4	40	100

a. **(4 points)** Fill the table above.

b. **(2 points)** Plot a percent frequency histogram.
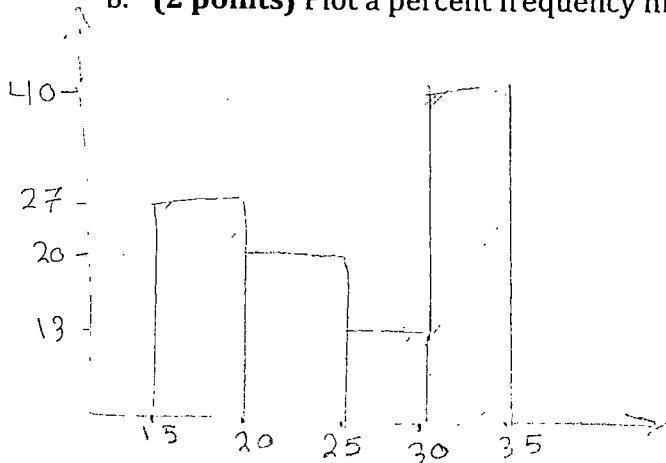
c. **(2 points)** Plot a cumulative percentage polygon (OGIVE).
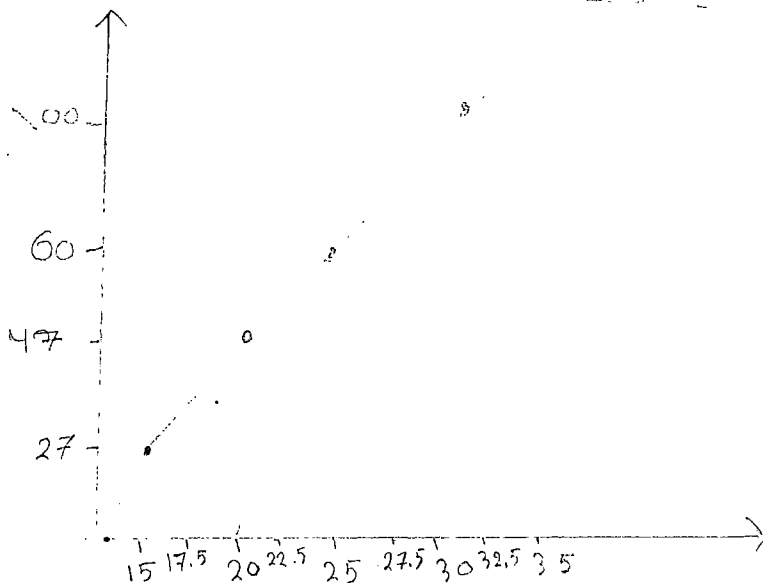
(84

4. Suppose that patrons of a restaurant were asked whether they preferred water or whether they preferred soda **(S)**. 70% of the patrons are males **(M)**. 15% of the females **(F)** preferred soda. 80% of the males **(M)** preferred water **(W)**.

    a. **(4 points)** Find the probability that a randomly selected patron prefers water.

    b. **(5 points)** If a patron is selected at random, find the probability that the patron is a male given that the patron prefers soda.

5. There are 8 instructors and 4 secretaries in a school.

   a. **(2 points)** How many different ways can you choose 3 instructors?

   b. **(2 points)** How many different ways can you choose 2 secretaries?

   c. **(3 points)** How many different ways can you choose 3 instructors and 2 secretaries?

   d. **(3 points)** How many different ways you can choose 3 instructors and 2 secretaries, if the youngest instructor and the youngest secretary have to be in the group?

88

Name: ...._Key_.........

GUST ID #: ....

50

MIDTERM

1. Given the following table.

Class	Cumulative frequency	Frequency	Relative Frequency	Percent frequency	Cumulative percent frequency
15 but less than 20	4	4	$\frac{4}{15} = 0.27$	27	27
20 but less than 25	7	3	$\frac{3}{15} = 0.20$	20	47
25 but less than 30	9	2	$\frac{2}{15} = 0.13$	13	60
30 but less than 35	15	6	$\frac{6}{15} = 0.40$	40	100

a. **(4 points)** Fill the table above.
b. **(2 points)** Plot a percent frequency histogram.

c. **(2 points)** Plot a cumulative percentage polygon (OGIVE).

(89

3. A sample of 300 men is selected. The contingency table below shows their registration status and their preferred source of information on current events. If an man is selected at random,

		Preferred source of information				
		Television	Newspapers	Radio	Internet	
Voting registration status	Registered	45	30	45	36	156
	Not registered	35	44	45	20	144
		80	74	90	56	300

a. **(2 points)** What is the probability that he prefers to get his current information from the newspapers?

$$\frac{74}{300}$$

b. **(2 points)** What is the probability that he is not a registered voter and prefers to get his current information from the internet?

$$\frac{20}{300}$$

c. **(3 points)** What is the probability that he is a registered voter or prefers to get his current information from the television?

$$\frac{45+30+45+36+35}{300} = \frac{191}{300}$$

d. **(3 points)** What is the probability that he is a registered voter given that he prefers to get his current information from the radio?

$$\frac{45}{90}$$

91

$n=6$

2. Given the data sample: 5, 9, 10, 7, 8, 9

$5, 7, 8, 9, 9, 10$

a. **(3 points)** Compute the mean, median and mode.

$$\text{mean} = \bar{x} = \frac{5+7+8+9+9+10}{6} = \frac{48}{6} = 8$$

$$\text{median} = \frac{6+1}{2} = 3.5 \Rightarrow \frac{3^{rd}+4^{th}}{2} = \frac{8+9}{2} = 8.5$$

$$\text{mode} = 9.$$

b. **(2 points)** Compute the standard deviation and the coefficient of variation.

$$s^2 = \frac{(5-8)^2 + (7-8)^2 + (8-8)^2 + (9-8)^2 + (9-8)^2 + (10-8)^2}{6-1} = \frac{9+1+0+1+1+4}{5}$$

$$s^2 = \frac{16}{5} \Rightarrow s = \sqrt{\frac{16}{5}} = 1.79$$

$$CV = \frac{s}{\bar{x}} \times 100 = \frac{1.79}{8} \times 100 = 22.4\%$$

c. **(2 points)** Find whether 16 an outlier or not.

$$\text{z-score} = \frac{16-8}{1.79} = 4.4773 \rightarrow \text{yes it is.}$$

d. **(4 points)** Compute the range, Q1, Q3 and IQR. Range $= \max - \min = 10-5 = 5$

Q1 position: $\frac{6+1}{4} = 1.75 \Rightarrow Q_1 = 2^{nd}$ data $\Rightarrow Q_1 = 7$

Q3 " $\cdot 3(\frac{6+1}{4}) = 5.25 \Rightarrow Q_3 = 5^{th}$ data $\Rightarrow Q_3 = 9$

$$IQR = Q_3 - Q_1 = 9-7 = 2$$

e. **(2 points)** Draw a box plot displaying the five number summary and comment on the shape of the distribution.

left skewed.

Q_1 Q_2

5   7   8   9

$\left(90\right.$

4. Suppose that patrons of a restaurant were asked whether they preferred water or whether they preferred soda (S). 70% of the patrons are males (M). 15% of the females (F) preferred soda. 80% of the males (M) preferred water (W).

    a. **(4 points)** Find the probability that a randomly selected patron prefers water.
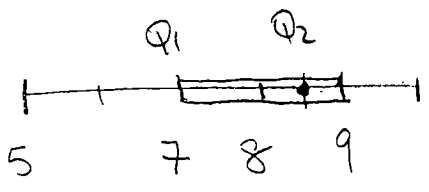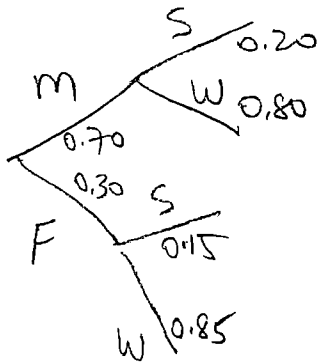
$P(\text{water}) = 0.70$

$= 0.70 \times 0.80 + 0.30 \times 0.85$

$= 0.56 + 0.255$

$= 0.815$

    b. **(5 points)** If a patron is selected at random, find the probability that the patron is a male given that the patron prefers soda.

$P(\text{Male} \mid \text{Soda}) = \dfrac{P(\text{Male} \cap \text{Soda})}{P(\text{Soda})} = \dfrac{0.70 \times 0.20}{0.185} = \dfrac{0.14}{0.185}$

$= 0.757$

$P(\text{Soda}) = 1 - P(\text{water}) = 1 - 0.815 = 0.185$

$\underline{\text{OR}} = 0.70 \times 0.20 + 0.30 \times 0.15 = 0.185$

92

5. There are 8 instructors and 4 secretaries in a school.

a. **(2 points)** How many different ways can you choose 3 instructors?

$$_8C_3 = 56$$

b. **(2 points)** How many different ways can you choose 2 secretaries?

$$_4C_2 = 6$$

c. **(3 points)** How many different ways can you choose 3 instructors and 2 secretaries?

$$_8C_3 \times {}_4C_2 = 56 \times 6 = 336$$

d. **(3 points)** How many different ways you can choose 3 instructors and 2 secretaries, if the youngest instructor and the youngest secretary have to be in the group?

$$_7C_2 \times {}_3C_1 = 63$$

$$21 \times 3 = 63$$

(93

Math 121 Basic Probability and Statistics

Fall 2012 Exam 2

Show all your work to get full credit.

Name:_____

_____

ID#:_____

1 (30 points) It is known that the probability of a power cut is 0.2 for a randomly selected day.

a) If you randomly select 12 days, what is the probability there will be power cut for 3 days?

b) Find the expected value and variance for the number of days with power cuts.

2) (30 points) A fisherman catches 3 big fish per day on average. What is the probability that he catches
a) at least 2 fish in one day?

b) 7 fish in two days?

3) (20 points) Delivery time for a pizza is uniformly distributed between 12 and 32 minutes. Graph the probability density function and find the probability that a customer waits between 25 and 35 minutes.

4) (20 points) The probability density function for X is

X	0	1	2
P(X)	0.2	0.6	0.2

a) Find the expected value, E(X).

b) Find the variance of X, Var(X) and the standard deviation of X

Bonus)

a) For the data set 12, 16, 17, 14, 15, 16   compute the quartiles Q1, Q2 and Q3.

b) How many different ways can you choose 4 instructors and 3 secretaries out of 8 instructors and 6 secretaries?

/95

Math 121 Basic Probability and Statistics

Fall 2012 Exam 2

Name:_____

_____

ID#:_____

Show all your work to get full credit.

1 (30 points) It is known that the probability of a power cut is 0.2 for a randomly selected day.

a) If you randomly select 12 days, what is the probability there will be power cut for 3 days?

$$P(X=3) = C_{12\ 3} \cdot (0.2)^3 (0.8)^9 = 0.2362$$

Binomial
$n = 12$

$p = 0.2$

$1-p = 0.8$

b) Find the expected value and variance for the number of days with power cuts.

$$E(X) = n \cdot p = 12 \times 0.2 = 2.4$$

$$Var(X) = np(1-p) = 12 \times 0.2 \times 0.8 = 1.92$$

2) (30 points) A fisherman catches 3 big fish per day on average. What is the probability that he catches

a) at least 2 fish in one day? $\rightarrow \lambda = 3$

Poisson ; $\lambda = 3$ per day

$$P(X \geq 2) = P(X=2) + P(X=3) + \cdots + P(X=\infty)$$

$$= 1 - \{P(X=0) + P(X=1)\} = 1 - \left\{ e^{-3} \frac{3^0}{0!} + e^{-3} \frac{3^1}{1!} \right\} = 1 - \{0.0497 + 0.1493\}$$

$$= 1 - 0.199$$
$$= 0.801$$

b) 7 fish in two days? new $\lambda = 2 \times 3 = 6$

$$P(X=7) = e^{-6} \cdot \frac{6^7}{7!} = 0.1377$$

3) (20 points) Delivery time for a pizza is uniformly distributed between 12 and 32 minutes. Graph the probability density function and find the probability that a customer waits between 25 and 35 minutes.

$$\rightarrow P(25 < X < 35) = 7 \times \frac{1}{20} = 0.35$$
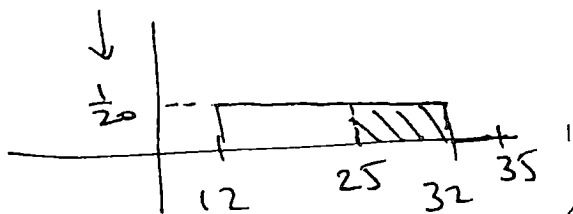
(96

4) (20 points) The probability density function for X is

X	0	1	2
P(X)	0.2	0.6	0.2

a) Find the expected value, E(X).

$$E(x) = 0 \times 0.2 + 1 \times 0.6 + 2 \times 0.2$$
$$= 0 + 0.6 + 0.4 = 1.$$

b) Find the variance of X, Var(X) and the standard deviation of X

$$Var(x) = (0-1)^2 \times 0.2 + (1-1)^2 \times 0.6 + (2-1)^2 \times 0.2$$
$$= 0.2 + 0 + 0.2 = 0.4$$

$$st \ dev = \sqrt{0.4} = 0.632$$

Bonus)

a) For the data set 12, 16, 17, 14, 15, 16   compute the quartiles Q1, Q2 and Q3.

$$12, 14, 15, 16, 16, 17 \qquad n = 6$$

$$Q1 \Rightarrow \frac{6+1}{4} = 1.75 \Rightarrow Q_1 = 2^{nd} \ data = 14$$

$$Q_2 \Rightarrow \frac{2(6+1)}{4} = 3.5 \Rightarrow Q_2 = \frac{3^{rd} data + 4^{th} data}{2} = \frac{15+16}{2} = 15.5$$

$$Q_3 \Rightarrow \frac{3(6+1)}{4} = \frac{21}{4} = 5.25 \Rightarrow Q_3 = 5^{th} data = 16$$

b) How many different ways can you choose 4 instructors and 3 secretaries out of 8 instructors and 6 secretaries?

$$_8C_4 \times _6C_3 = 70 \times 20 = 1400$$

97

Gulf University for Science and Technology
College of Arts and Sciences

Basic Probability and Statistics

Final Examination – FALL 2012

Course Code: MATH 121   Section:

Instructor: Date: January 5th , 2013
(Please circle your instructor's name)

Harun Aydilek	Helmi Temimi	Mohamed B. Romdhane	Munir Mahmood

Student Name: _____

Student Number: _____

Aids Allowed You can use a calculator, but it cannot be shared.
Instructions:

1. The exam is double sided.
2. There are 2 tables for Normal distribution.
3. Formulas and tables are attached to the exam. Do not separate them.

This examination has a cover page, 10 pages with 9 questions, 2 pages for formula sheets, 2 pages for tables. Total pages is 15 counting double side. Before you start the examination please verify them.

No Questions are allowed during the examination
Student signature: _____

Objective	1	2	3	4	5		6		7	Total
Question	1	2	3	4	5	6	7	8	9	
Score per question										
Score per objective										
Out of	13	16	13	12	19		17		10	100

$\left(98 \right.$

1)

a. **(5 points)** Complete the following table:

Classes	Frequency	Cumulative Frequency	Percentage	Cumulative percentage
00 but less than 10	4			
10 but less than 20		10		
20 but less than 30	8			
30 but less than 40		20		
Total				

b. **(4 points)** Make a histogram of **percentages**.

c. **(4 points)** Make a cumulative percentage polygon (OGIVE).

2

(99

2) Given the following set of numbers:

$$2, 3, 4, 8, 10, 15$$

a) Find the:

    i. **(2 point)** Sample mean $\bar{x}$

    ii. **(2 point)** Median

    iii. **(2 points)** Sample Variance s^2

    iv. **(1 point)** Sample Standard Deviation s

    v. **(1 point)** Coefficient of Variation CV

b) (3 points) Find Q_1 and Q_3.

c) (2 points) Make a **Boxplot**, displaying the five numbers summary and conclude on **the type of the shape** of the distribution.

d) (2 points) Find the **Z-score** for the grade 17. Is this an **outlier, why?**

e) (1 point) If the data set was 9, 9, 9, 9, 9, 9; what would be the **standard deviation?**

4) There are 7 finalists at a national swimming competition.

   a. **(4 points)** In how many different ways can the 7 finalists finish the race?

   b. **(3 points)** If only three of them on the podium, how many different orders can we see on the podium?

   c. **(5 points)** If the organizing committee decides to select randomly four of the swimmers to participate in a different competition, how many different ways can they select this group of four swimmers?

3) The following table shows the number of students in three different degree programs and whether they are graduate or undergraduate students.

	Undergraduate	Graduate	Total
Business	150	50	200
Engineering	150	25	175
Sciences	100	25	125
Total	400	100	500

a. **(3 points)** What is the probability that a randomly selected student is an undergraduate?

b. **(4 points)** What is the probability that a randomly selected student is engineering major and graduate?

c. **(4 points)** Given that the selected student is an undergraduate, what is the probability that this student is a business major?

d. **(2 points)** What is the probability that a randomly selected student is a business or engineering major?

5

102

5) In a basic course of Mathematics, **seventy percent (70%)** of the students pass the course. Assuming a **binomial** probability distribution model, and given that the total number of students in this course is 8

    a. **(3 points)** What is the probability that NO student will pass the course?

    b. **(4 points)** What is the probability that at least 2 students will pass the course?

    c. **(3 points)** Determine the expected number of students passing the course and compute the standard deviation.

6) In a computer network, the average of network communication errors is **2.5 per hour**. Assuming a **Poisson** probability distribution model, find:

    a. **(3 points)** the probability that there will be exactly **one** communication error in the **next one hour**?

    b. **(3 points)** the probability that there will be exactly **three** communication errors in the **next two hours**?

    c. **(3 points)** the probability that there will be at **most two** communication errors in the **next one hour**?

7) The length of time it takes to drive a trip from Ahmadi to Kuwait City is **uniformly** distributed and varies between 40 and 50 minutes.

    a) **(1 points)** Graph the probability density function.

    b) **(3 points)** Compute the probability that a student will take between 45 and 70 minutes to complete the trip.

    c) **(2 points)** Compute the probability that a student will take no more than 40 minutes to complete the trip.

    d) **(2 points)** What is the expected amount of time it takes to complete the trip?

9

/10 6

8) The weights of the contents of cans of pasta sauce produced by a company are **normally** distributed with a mean of 7 ounces and a standard deviation of 3 ounces.

   a. **(3 points)** If we randomly select a can, what is the probability that it contains more than 7.45 ounces of pasta sauce?

   b. **(3 points)** If we randomly select a can, what is the probability that it contains between 6.4 and 8.8 ounces of pasta sauce?

   c. **(3 points)** Ninety-five percent (95%) of cans will contain at least how many ounces of pasta sauce (Hint: what is w such that $P(X \geq w) = 0.95$)?

10

107

9) The weights of the contents of cans of pasta sauce produced by a company are **normally** distributed with a mean of 8 ounces and a standard deviation of 3 ounces. A **random sample of 4** cans is selected.

    a. **(4 points)** Find the standard error of the sample weight mean?

    b. **(6 points)** What is the probability that the sample weight mean is between 6.5 and 9.5 ounces?

11

Gulf University for Science and Technology
College of Arts and Sciences

Basic Probability and Statistics

Final Examination – FALL 2012

Course Code: MATH 121   Section:

Instructor: Date: January 5th , 2013
(Please circle your instructor's name)

Harun Aydilek	Helmi Temimi	Mohamed B. Romdhane	Munir Mahmood

Student Name: _____Key_____

Student Number: _____

Aids Allowed You can use a calculator, but it cannot be shared.
Instructions:

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3. **Formulas and tables are attached to the exam. Do not separate them.**

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Student signature: _____

Objective	1	2	3	4	5		6		7	Total
Question	1	2	3	4	5	6	7	8	9	
Score per question										
Score per objective										
Out of	13	16	13	12	19		17		10	100

1

Q1)

a. **(5 points)** Complete the following table:

Classes	Frequency	Cumulative Frequency	Percentage	Cumulative percentage
00 but less than 10	4	4	$4/20 \times 100 = 20$	20
10 but less than 20	6	10	= 30	50
20 but less than 30	8	18	= 40	90
30 but less than 40	2	20	= 10	100
Total	20		100	

b. **(4 points)** Make a histogram of **percentages**.
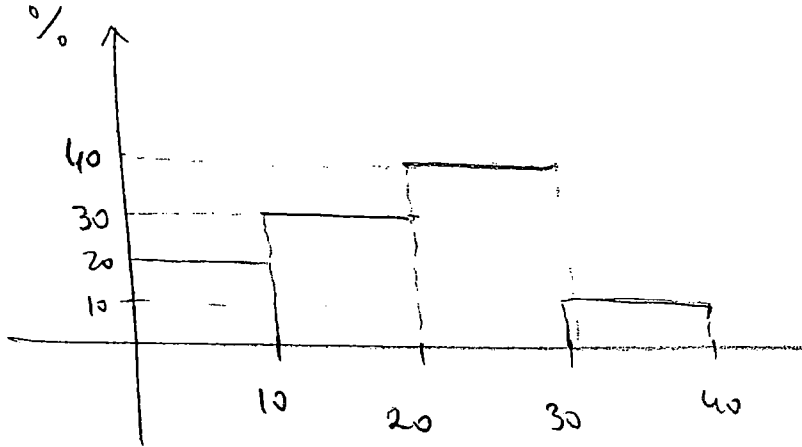
c. **(4 points)** Make a cumulative percentage **polygon (OGIVE).**

OGIVE

4

(110
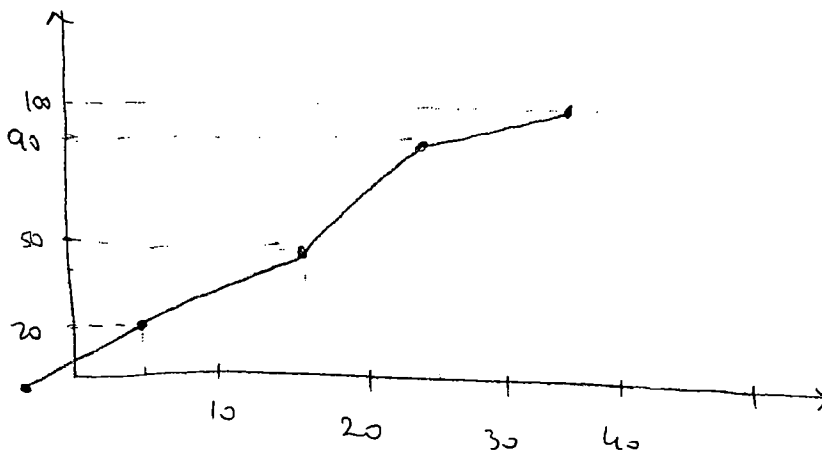
q 2) Given the following set of numbers:

$$2, 3, 4, 8, 10, 15 \qquad n = 6$$

a) Find the:

i. **(2 point)** Sample mean $\bar{x}$

$$\bar{x} = \frac{2+3+4+8+10+15}{6} = \frac{42}{6} = 7$$

ii. **(2 point)** Median

$$\text{median} = \frac{4+8}{2} = 6$$

iii. **(2 points)** Sample Variance s^2

$$s^2 = \frac{(2-7)^2 + (3-7)^2 + (4-7)^2 + (8-7)^2 + (10-7)^2 + (15-7)^2}{6-1}$$

$$= \frac{25+16+9+1+9+64}{5} = \frac{124}{5} = 24.8$$

iv. **(1 point)** Sample Standard Deviation s

$$S = \sqrt{24.8} = 4.98$$

v. **(1 point)** Coefficient of Variation CV

$$CV = \frac{s}{\bar{x}} \times 100 = \frac{4.98}{7} \times 100 = 71.14\%$$

2

(11)

b) (3 points) Find Q_1 and Q_3.

$$Q_1 \Rightarrow \frac{6+1}{4} = 1.75 \Rightarrow Q_1 = 2^{nd} \ data = 3$$

$$Q_3 = 3\left(\frac{6+1}{4}\right) = 5.25 \Rightarrow Q_1 = 5^{th} \ data = 10$$

c) (2 points) Make a **Boxplot**, displaying the five numbers summary and conclude on the **type of the shape** of the distribution.
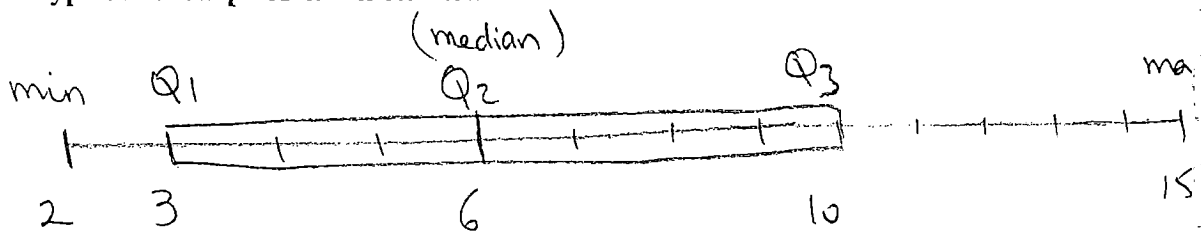
Right skewed.

d) (2 points) Find the **Z-score** for the grade 17. Is this an **outlier**, why?

$$Z = \frac{x - \bar{x}}{s} = \frac{17 - 7}{4.98} = \frac{10}{4.98} = 2 < 3 \Rightarrow \text{Not an outlier.}$$

e) (1 point) If the data set was 9, 9, 9, 9, 9, 9; what would be the **standard deviation**?

$$S = 0.$$

3

(112

3) The following table shows the number of students in three different degree programs and whether they are graduate or undergraduate students.

	Undergraduate	Graduate	Total
Business	150	50	200
Engineering	150	25	175
Sciences	100	25	125
Total	400	100	500

a. **(3 points)** What is the probability that a randomly selected student is an undergraduate?

$$\frac{400}{500} = 0.8$$

b. **(4 points)** What is the probability that a randomly selected student is engineering major and graduate?

$$\frac{25}{500} = 0.05$$

c. **(4 points)** Given that the selected student is an undergraduate, what is the probability that this student is a business major?

$$\frac{150}{400} = 0.375$$

d. **(2 points)** What is the probability that a randomly selected student is a business or engineering major?

$$\frac{375}{500} = 0.75$$

5

/113

Q4) There are 7 finalists at a national swimming competition.

   a. **(4 points)** In how many different ways can the 7 finalists finish the race?

$$7! = 5040$$

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

   b. **(3 points)** If only three of them on the podium, how many different orders can we see on the podium?

$$\underline{7} \times \underline{6} \times \underline{5} = 210$$

or

$$_7P_3 = 210.$$

   c. **(5 points)** If the organizing committee decides to select randomly four of the swimmers to participate in a different competition, how many different ways can they select this group of four swimmers?

$$_7C_4 = 35$$

(/14

Q5) In a basic course of Mathematics, **seventy percent** (70%) of the students pass the course. Assuming a **binomial** probability distribution model, and given that the total number of students in this course is 8

    a. **(3 points)** What is the probability that NO student will pass the course?

$$P(X=0) = {}_8C_0 (0.7)^0 (0.3)^8 =$$

$$= 6.56 \times 10^{-5}$$

$$= 0.0000656$$

pass $\to 0.70$

fail $\to 0.30$
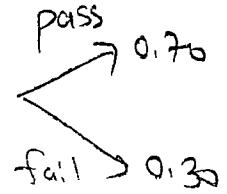
$n = 8$

    b. **(4 points)** What is the probability that at least 2 students will pass the course?

$$P(X \geq 2) = P(X=2) + \ldots + P(X=8)$$

$$= 1 - \{ P(X=0) + P(X=1) \}$$

$${}_8C_1 (0.7)^1 (0.3)^7$$

$$= 1 - \{ 0.0000656 + 0.00122 \}$$

$$= 0.9987144$$

    c. **(3 points)** Determine the expected number of students passing the course and compute the standard deviation.

$$E(X) = np = 8 \times 0.7 = 5.6$$

$$V(X) = np(1-p) = 8 \times 0.7 \times 0.3 = 1.68$$

$$S.t \, dev = \sqrt{1.68} = 1.296$$

6

$\lambda = 2.5$ per hour

6) In a computer network, the average of network communication errors is **2.5 per hour**. Assuming a Poisson probability distribution model, find:

a. **(3 points)** the probability that there will be exactly **one** communication error in the **next one hour?**

$\lambda = 2.5$

$$P(x=1) = e^{-2.5}\frac{2.5^1}{1!} = 0.2052$$

b. **(3 points)** the probability that there will be exactly **three** communication errors in the **next two hours?**

$\lambda = 2.5 \times 2 = 5$

$$P(y=3) = e^{-5}\frac{5^3}{3!} = 0.1404$$

c. **(3 points)** the probability that there will be at **most two** communication errors in the **next one hour?**

$\lambda = 2.5$

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= e^{-2.5}\frac{2.5^0}{0!} + e^{-2.5}\frac{2.5^1}{1!} + e^{-2.5}\frac{2.5^2}{2!}$$
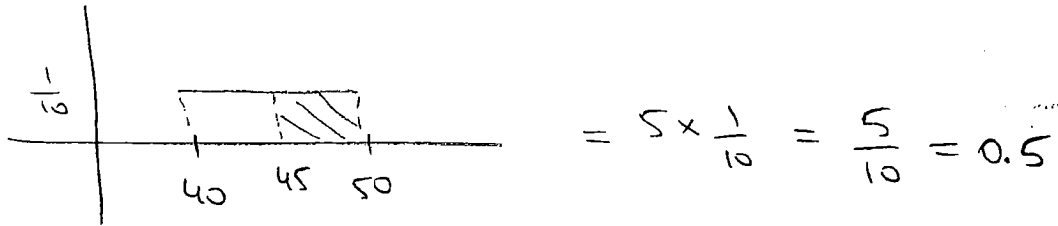
$$= 0.5438$$

8

$(116$

Q7) The length of time it takes to drive a trip from Ahmadi to Kuwait City is **uniformly** distributed and varies between 40 and 50 minutes.

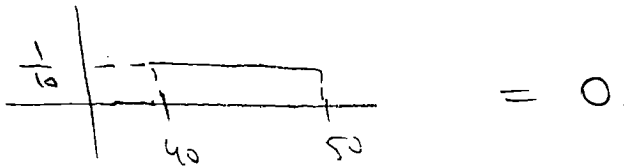a) **(1 points)** Graph the probability density function.
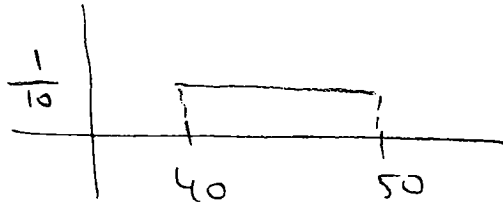
b) **(3 points)** Compute the probability that a student will take between 45 and 70 minutes to complete the trip.

$$P(45 < X < 70)$$

$$= 5 \times \frac{1}{10} = \frac{5}{10} = 0.5$$

c) **(2 points)** Compute the probability that a student will take no more than 40 minutes to complete the trip.

$$P(X \le 40)$$

$$= 0.$$

d) **(2 points)** What is the expected amount of time it takes to complete the trip?
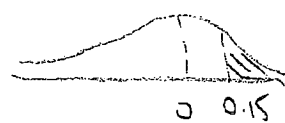
$$E(X) = \frac{40+50}{2} = 45 \text{ min}.$$

(117

$\mu = 7, \sigma = 3$

Q 8) The weights of the contents of cans of pasta sauce produced by a company are **normally** distributed with a mean of 7 ounces and a standard deviation of 3 ounces.

    a. **(3 points)** If we randomly select a can, what is the probability that it contains more than 7.45 ounces of pasta sauce?

$$P(X > 7.45) = P\left(\frac{X - \mu}{\sigma} > \frac{7.45 - \mu}{\sigma}\right)$$

$$= P\left(Z > \frac{7.45 - 7}{3}\right) = P(Z > 0.15) =$$

$$1 - = 1 - 0.5596 = 0.4404$$

    b. **(3 points)** If we randomly select a can, what is the probability that it contains between 6.4 and 8.8 ounces of pasta sauce?
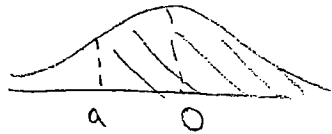
$$P(6.4 < X < 8.8) = P\left(\frac{6.4 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{8.8 - \mu}{\sigma}\right)$$

$$= P\left(\frac{6.4 - 7}{3} < Z < \frac{8.8 - 7}{3}\right) = P(-0.2 < Z < 0.6)$$

$$= 0.7257 - 0.4207 = 0.3050$$

    c. **(3 points)** Ninety-five percent (95%) of cans will contain at least how many ounces of pasta sauce (Hint: what is w such that $P(X \geq w) = 0.95$)?

$$\Rightarrow \text{ look for } a \text{ s.t. } P(Z \geq a) = 0.95$$

$$\Rightarrow 1 - 0.95 = 0.05 \Rightarrow (-) \text{ table}$$

$$\Rightarrow a = -1.64 \text{ (or } -1.65) \Rightarrow w = (a \cdot \sigma) + \mu$$

$$\Rightarrow w = (-1.64) \cdot (3) + 7$$

$$w = -4.92 + 7 = 2.08$$

9

(118

$\mu = 8, \sigma =$

9) The weights of the contents of cans of pasta sauce produced by a company are **normally** distributed with a mean of 8 ounces and a standard deviation of 3 ounces. A **random sample of 4 cans is** selected.
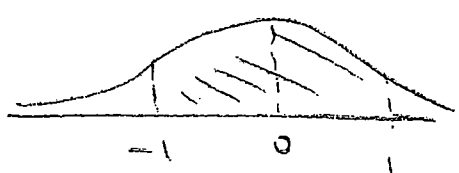
      a. **(4 points)** Find the standard error of the sample weight mean?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{4}} = \frac{3}{2} = 1.5$$

      b. **(6 points)** What is the probability that the sample weight mean is between 6.5 and 9.5 ounces?

$$P(6.5 < \bar{x} < 9.5) = P\left(\frac{6.5 - \mu}{\sigma_{\bar{x}}} < \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{9.5 - \mu}{\sigma_{\bar{x}}}\right)$$

$$= P\left(\frac{6.5 - 8}{1.5} < z < \frac{9.5 - 8}{1.5}\right)$$

$$= P(-1 < z < 1) =$$

$$= 0.8413 - 0.1587 = 0.6826$$

9

(119